582206 Models of Computation (Autumn 2009)
Exercise 4 (29 September – 2 October)

Basic exercises

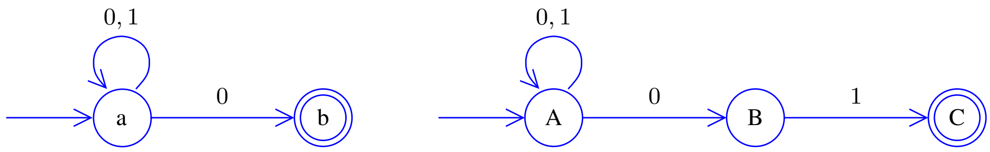
The first three problems are basic applications of the material from the text book. Solve them by yourself; if there is anything unclear you can ask about it during the exercise session.

1. Give a state diagram for the NFA $N = (Q, \Sigma, \delta, q_0, F)$ where $Q = \{ q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7 \}$, $\Sigma = \{ \mathrm{a}, \mathrm{b}, \mathrm{c} \}$, $F = \{ q_4 \}$ and δ is as follows:

δ	a	b	c	ε
q_0	$\emptyset$	$\emptyset$	$\emptyset$	$\{ q_1, q_5 \}$
q_1	$\{ q_1, q_2 \}$	$\{ q_1 \}$	$\{ q_1 \}$	$\emptyset$
q_2	$\emptyset$	$\{ q_3 \}$	$\emptyset$	$\{ q_3 \}$
q_3	$\emptyset$	$\emptyset$	$\{ q_4 \}$	$\emptyset$
q_4	$\{ q_4 \}$	$\{ q_4 \}$	$\{ q_4 \}$	$\emptyset$
q_5	$\{ q_5 \}$	$\{ q_5 \}$	$\{ q_5, q_6 \}$	$\emptyset$
q_6	$\emptyset$	$\{ q_7 \}$	$\emptyset$	$\{ q_7 \}$
q_7	$\{ q_4 \}$	$\emptyset$	$\emptyset$	$\emptyset$

   What is the language recognised by the NFA?

2. Let the language A over alphabet $\{ 0, 1 \}$ consist of string that end in zero. Further, let the language B consist of strings that end in 01. The languages A and B can be recognized with the following NFAs:



   (a) Construct an NFA for the language $A \circ B$ using the construction from lectures (ss. 82–83, Sipser ss. 60–61). Give also the formal description of the NFA.

   (b) Show the computation tree (lectures s. 62, Sipser s. 49) for the input 001101.

   (c) Construct a DFA for the same language using the construction from lectures (ss. 72–74, Sipser ss. 55–56).

3. Show that the class of regular languages is closed under the reversal operation defined in Problem 1 of Exercise 3. *Hint:* Modify the construction of Problem 3.1 so that it works for multiple final states, too.

Continues on the next page!

Discussion problems

Read the following problems and make sure you are familiar with the necessary basic concepts. You are not expected to solve the problems by yourself; we shall discuss them together.

4. What is wrong with the following proof.

   Let L be a language. Clearly, for all $x, y, z \in L$, the sets $\{ x \}$, $\{ y \}$, $\{ z \}$ are regular languages. Because the class of regular languages is closed under union, the sets $\{ x, y \} = \{ x \} \cup \{ y \}$ and $\{ xyz \} = \{ xy \} \cup \{ z \} = \{ x \} \cup \{ y \} \cup \{ z \}$ are regular, too. This proof generalizes for sets of arbitrarily many members. Thus
   $$L = \bigcup_{w \in L} \{ w \}$$
   is regular, too.

5. Show that the class of regular languages is closed

   (a) under complement and

   (b) under intersection.

   Here it is easier to use constructions based on deterministic automata. Try to use constructions based on nondeterministic automata and report what difficulties you encounter.

6. Let the language A_n over alphabet $\{ \texttt{a}, \texttt{b} \}$ consist of strings where the nth symbol from the end is $\texttt{a}$.

   (a) Construct an NFA recognizing A_n with at most $n + 1$ states.

   (b) Show that any DFA recognizing A_n must have at least 2^n states.