582206 Models of Computation (Autumn 2009)
Exercise 5 (6–9 October)

1. (a) We say that a string w is a *prefix* of a string x, if a string z exists such that $x = wz$. For any language A over alphabet Σ, we define its set of prefixes as

$$\text{PREFIX}(A) = \{\, w \in \Sigma^* \mid \text{there exists } z \in \Sigma^* \text{ such that } wz \in A \,\}\,.$$

Prove that if A is regular, then so is $\text{PREFIX}(A)$.

   (b) We say that a string w is a *suffix* of a string x, if a string z exists such that $x = zw$. For any language A, we define its set of suffixes as

$$\text{SUFFIX}(A) = \{\, w \in \Sigma^* \mid \text{there exists } z \in \Sigma^* \text{ such that } zw \in A \,\}\,.$$

Prove that if A is regular, then so is $\text{SUFFIX}(A)$. *Hint*: you may apply part (a) together with the result from Problem 3 of Exercise 4.

2. Give a regular expression for each of the following languages over the alphabet $\Sigma = \{\,0, 1\,\}$:

   (a) strings that contain 000 or 111 as substring
   (b) strings that contain both 000 and 111 as substring
   (c) strings where the last two characters are the same (and in same order) as the first two
   (d) strings that do not contain 000 as substring.

3. Define a *comment* as a string that begin with the two characters "`/*`", ends with the two characters "`*/`" and does not contain a "`*/`" combination otherwise. For simplicity we consider comments consisting of only characters '`a`', '`b`', '`*`' and '`/`'. Give a (a) DFA (b) regular expression for the language that consists of all comments.

4. Convert the following DFA into a regular expression using the method given in Lemma 1.60 of Sipser's book: