582206 Models of Computation (Autumn 2009)
Exercise 6 (13 – 16 October)

Basic exercises

The first three problems are basic applications of the material from the text book. Solve them by yourself; if there is anything unclear you can ask about it during the exercise session.

1. For each of the languages, give two strings that are members and two strings that are not members. Assume the alphabet $\Sigma = \{ a, b \}$ in all cases.

   (a) a^*b^*

   (b) $a(ba)^*b$

   (c) $a^* \cup b^*$

   (d) $(aaa)^*$

   (d) $\Sigma^*a\Sigma^*b\Sigma^*a\Sigma^*$

   (e) $aba \cup bab$

   (f) $(\varepsilon \cup a)b$

   (g) $(a \cup ba \cup bb)\Sigma^*$

2. Which of the following statements are true and which are false? For each statement, justify your answer by giving a proof or a counter-example.

   (a) If $A \cup B$ is regular and A is regular, then B is regular.

   (b) If $A \cup B$ is not regular and B is regular, then A is not regular.

   (c) If A is regular and B is not regular, then $A \cup B$ is not regular.

3. (Sipser Problem 1.55) A natural number p is a *pumping length* for a language A if any string $w \in A$ with $|w| \geq p$ can be pumped (as in the statement of the Pumping lemma). The *minimum pumping length* for A is the smallest p such that p is a pumping length for A. For each of the following languages, determine its minimum pumping length and justify your answer.

   $C_1 = (01)^*$

   $C_2 = 1^*01^*01^*$

   $C_3 = \varepsilon$

   $C_4 = 010 \cup 00100.$

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Discussion problems

Read the following problems and make sure you are familiar with the necessary basic concepts. You are not expected to solve the problems by yourself; we shall discuss them together.

4. Which of the following languages over the alphabet $\Sigma = \{\, 0, 1 \,\}$ are regular?

$$A_1 = \{\, 0^n 1 0^n \mid n \in \mathbb{N} \,\} \qquad A_3 = \left\{\, w w^{\mathcal{R}} \mid w \in \Sigma^* \,\right\}$$
$$A_2 = \{\, 0^n 0^n \mid n \in \mathbb{N} \,\}. \qquad A_4 = \left\{\, w u w^{\mathcal{R}} \mid w, u \in \Sigma^+ \,\right\}$$

   Justify your answers *e.g.* by giving a finite automaton or applying the pumping lemma.

5. Prove that the following languages over the alphabet $\Sigma = \{\, 0, 1 \,\}$ are not regular:

$$\begin{aligned} B_1 &= \left\{\, 0^m 1^k 0^n \mid m, k, n \in \mathbb{N} \text{ and } m \neq n \,\right\} \\ B_2 &= \left\{\, w \in \Sigma^* \mid w \neq w^{\mathcal{R}} \,\right\}. \end{aligned}$$

   You may use the results from the previous problem and the closure properties of regular languages.

6. Let X be a non-regular language in the alphabet $\Sigma = \{\, a, b \,\}$. Define the following languages in the same alphabet:

$$\begin{aligned} &A = \{\, aba \,\} \circ \Sigma^* \\ &B = \overline{A} \\ &C = (\{\, aba \,\} \circ X) \cup B \end{aligned}$$

   Show that the language C

   (a) satisfies the conditions of the Pumping lemma (Hint: choose $p = 3$.)

   (b) is not regular. (Hint: consider the language $D = A \cap C$.)