582206 Models of Computation (Autumn 2009)
Exercise 7 (3 – 6 November)

Basic exercises

The first three problems are basic applications of the material from the text book. Solve them by yourself; if there is anything unclear you can ask about it during the exercise session.

1. Consider the context-free grammar

$$\begin{aligned} S &\rightarrow SAB \mid \varepsilon \\ A &\rightarrow \mathrm{a}A \mid \mathrm{a} \\ B &\rightarrow \mathrm{b}B \mid \varepsilon \end{aligned}$$

   Give a derivation and a parse tree for the string abbaab. What is the language of the grammar?

2. Give context-free grammars for the following languages over the alphabet $\Sigma = \{0, 1\}$:
   (a) all strings
   (b) strings that begin with 0
   (c) strings containing 111 as substring
   (d) strings with at least two characters where the first and the last character are the same
   (e) strings of odd length
   (f) $01^* \cup 10^*$

3. Show that the class of context-free languages is closed under the operations concatenation and star.

Discussion problems

Read the following problems and make sure you are familiar with the necessary basic concepts. You are not expected to solve the problems by yourself; we shall discuss them together.

4. Consider the context-free grammar of Problem 1. Give two different parse trees and the corresponding derivations for the string aa.

5. Give context-free grammars for the following languages over the alphabet $\Sigma = \{0, 1\}$:
   (a) $\{0^n1^m \mid m, n \in \mathbb{N} \text{ and } m \geq n\}$
   (b) $\left\{0^n1^k0^m \mid m, n, k \in \mathbb{N} \text{ and } k = n + m\right\}$
   (c) strings where there are an equal number of zeros and ones
   (d) strings of odd length where the last character and the middle character are the same.

6. A context-free grammar is called *right-regular*, if all the rules are of the form $A \rightarrow \varepsilon$, $A \rightarrow a$ or $A \rightarrow aB$, where A and B are variables and a is a terminal.
   (a) Show that the language of a right-regular grammar is regular. (*Hint*: construct an NFA where the states correspond to the variables of the grammar.)
   (b) Show that any regular language can be generated by a right-regular grammar.