582206 Models of Computation (Autumn 2009)
Exercise 9 (17 – 20 November)

Basic exercises

The first three problems are basic applications of the material from the text book. Solve them by yourself; if there is anything unclear you can ask about it during the exercise session.

1. Given a context-free grammar G, how can you decide whether $L(G) \neq \emptyset$? *Hint:* Exercise 8, Problem 3.
2. Consider the Chomsky normal form grammar

$$\begin{aligned} S &\to AX \mid AY \mid \mathrm{a} & \qquad A &\to \mathrm{a} \\ X &\to AX \mid \mathrm{a} & B &\to \mathrm{b} \\ Y &\to BY \mid \mathrm{a} \quad . \end{aligned}$$

   Apply the algorithm given on page 267 of the textbook to decide whether the string bbba is in the language generated by the grammar. Repeat with string abbba.
3. Give pushdown automata for the following languages over the alphabet $\Sigma = \{ a, b \}$:
   (a) strings of odd length
   (b) all palindromes
   (c) $\{ \mathrm{a}^i \mathrm{b}^j \mid 0 \leq i \leq j \}$

Discussion problems

Read the following problems and make sure you are familiar with the necessary basic concepts. You are not expected to solve the problems by yourself; we shall discuss them together.

4. Give an algorithm that receives as input a context-free grammar G and outputs the shortest string in $L(G)$. If there are several equally short strings in the language, it does not matter which one you choose.
5. Give pushdown automata for the following languages:
   (a) $\{ w \in \{ a, b \}^* \mid w$ has different number of a's than b's $\}$
   (b) $B = \{ \mathrm{a}^i \mathrm{b}^j \mathrm{c}^k \mid j = i + k \}$
   (c) strings of odd length over the alphabet $\Sigma = \{ a, b \}$ where the last character and the middle character are the same.
6. [Sipser Problem 2.35] Suppose a context-free grammar G is in Chomsky normal form and has b variables. Prove that if some string $w \in L(G)$ has a derivation of lenght at least 2^b, then $L(G)$ is infinite.