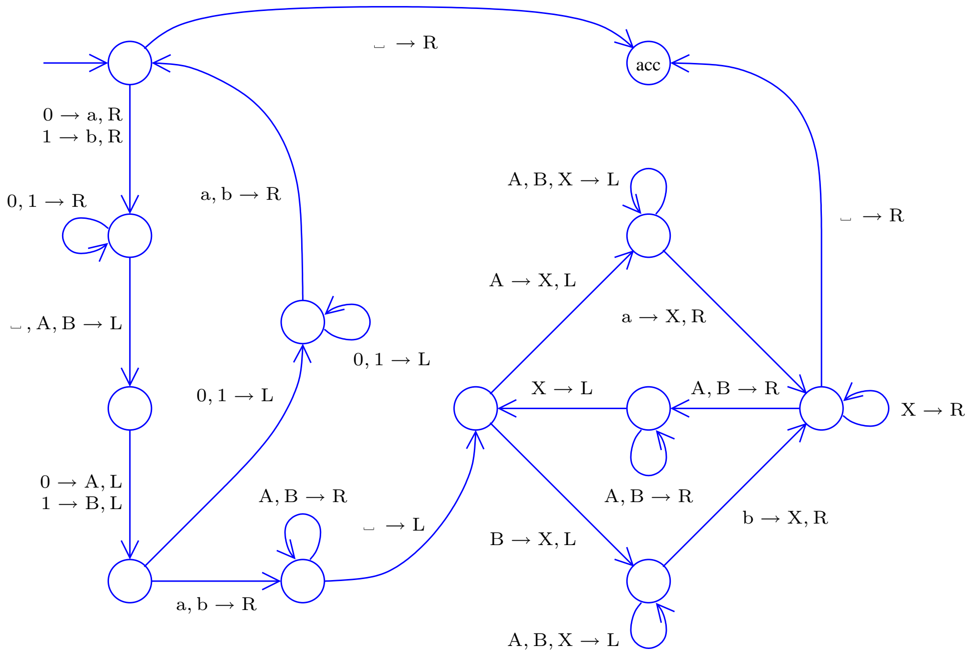
582206 Models of Computation (Autumn 2009)
Exercise 11 (1 – 4 December)

Basic exercises

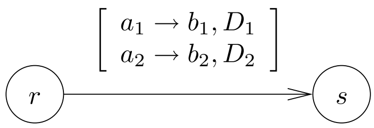
The first three problems are basic applications of the material from the text book. Solve them by yourself; if there is anything unclear you can ask about it during the exercise session.

1. Below is a Turing machine for the language $\{ ww \mid w \in \{0,1\}^* \}$. Show its computation (as a list of configurations) for the inputs 001001 and 101001.



2. Give a state diagram for a Turing machine that recognizes the language $\{ \mathrm{a}^i \mathrm{b}^j \mathrm{c}^i \mathrm{d}^j \mid i, j \in \mathbb{N} \}$.

3. Give a state diagram for a two-tape Turing machine that recognizes the language $\{ \mathrm{a}^n \mathrm{b}^n \mathrm{c}^n \mid n \in \mathbb{N} \}$. One suitable way of representing the transition $\delta(r, a_1, a_2) = (s, b_1, b_2, D_1, D_2)$ is



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Discussion problems

Read the following problems and make sure you are familiar with the necessary basic concepts. You are not expected to solve the problems by yourself; we shall discuss them together.

4. [Sipser Exercise 3.14] A *queue automaton* is like a push-down automaton, except that a queue replaces the stack. Two kinds of operations can be performed on the queue:

   - ENQUEUE(a) inserts the symbol a to the end of the queue
   - DEQUEUE reads and deletes the first symbol of the queue.

   As with a PDA, the input can be read one symbol at a time. We assume that the end of the input is marked with the symbol $\sqcup$ that may not appear anywhere else in the input. The queue automaton accepts by entering a special accept state (like a Turing machine).

   Show that any Turing-recognizable language can be recognized by a queue automaton. A sufficient solution to this problem is to explain, using a suitable level of pseudocode, how a Turing machine can be simulated using a queue.

5. [Part of Sipser Problems 3.15 and 3.16]

   (a) Show that the class of decidable languages is closed under union, intersection and complementation.

   (b) Show that the class of recognisable languages is closed under union and intersection. Why can't you use the same construction as in part (a) for complementation?

   Hint: similar construction as in the proof of Theorem 1.25, pp. 45–47.

6. [Sipser Exercise 3.4] Give a formal definition of an enumerator (see page 154). Consider it to be a type of two-tape Turing machine that uses its second tape as the printer. Include a definition of the enumerated language.