Algorithms for Bioinformatics (Autumn 2014)

Exercise 5 (Tue 07.10, 10-12, B222)

1. Ultrametric and additive distances.
   a) Which of the matrices below are ultrametric?
   b) Which of the matrices below are additive?

   \[
   \begin{array}{cccc|cccc|cccc}
   A & 0 & 4 & 10 & 10 & A & 0 & 8 & 7 & 4 & A & 0 & 4 & 6 & 3 \\
   B & 0 & 10 & 10 & & B & 0 & 5 & 6 & & B & 0 & 4 & 6 \\
   C & 0 & 8 & & & C & 0 & 5 & & & C & 0 & 7 \\
   D & 0 & & D & 0 & & D & 0 & & & & & & \\
   \end{array}
   \]

2. UPGMA.
   Simulate the UPGMA algorithm with the distance matrix given below. Check that the distances given by the resulting tree correspond to the distance matrix.

   \[
   \begin{array}{cccccc|cccc}
   & A & B & C & D & E & A & B & C & D & E \\
   A & 0 & 6 & 10 & 10 & 6 & A & 0 & 8 & 7 & 4 & A & 0 & 4 & 6 & 3 \\
   B & 0 & 10 & 10 & 2 & & B & 0 & 5 & 6 & & B & 0 & 4 & 6 \\
   C & 0 & 4 & 10 & & & C & 0 & 5 & & & C & 0 & 7 \\
   D & 0 & 10 & & & & D & 0 & & & & D & 0 \\
   E & 0 & & & & & & & & & & & & \\
   \end{array}
   \]

   Simulate the neighbor joining method with the distance matrix given below. Check that the distances given by the resulting tree correspond to the distance matrix.

   \[
   \begin{array}{cccc|cccc}
   & A & B & C & D & A & B & C & D \\
   A & 0 & 4 & 4 & 5 & A & 0 & 8 & 7 & 4 & A & 0 & 4 & 6 & 3 \\
   B & 0 & 6 & 3 & & B & 0 & 5 & 6 & & B & 0 & 4 & 6 \\
   C & 0 & 7 & & & & C & 0 & 5 & & & C & 0 & 7 \\
   D & 0 & & D & 0 & & D & 0 & & & & & & \\
   \end{array}
   \]

4. Three-point condition.
   Consider a symmetric distance matrix \( D = \{d_{ij} \mid 1 \leq i \leq n, 1 \leq j \leq n \} \). Show that the following two statements of the three-point condition are equivalent:

   - For all \( i, j, k \), \( d_{ij} \leq \max(d_{ik}, d_{kj}) \).
   - For all \( i, j, k \), two of the values \( d_{ij}, d_{ik}, \) and \( d_{kj} \) are equal and the third one is smaller than or equal to the others.

5. Ultrametric condition.
   Consider a symmetric distance matrix \( D = \{d_{ij} \mid 1 \leq i \leq n, 1 \leq j \leq n \} \) defined by an ultrametric tree. Show that \( D \) satisfies the three-point condition (see Problem 4 above).