Algorithms for Bioinformatics (Autumn 2014)

Exercise 5 (Tue 07.10, 10-12, B222)

1. **Ultrametric and additive distances.**

   a) Which of the matrices below are ultrametric?

   b) Which of the matrices below are additive?

	A	B	C	D
A	0	4	10	10
B		0	10	10
C			0	8
D				0

	A	B	C	D
A	0	8	7	4
B		0	5	6
C			0	5
D				0

	A	B	C	D
A	0	4	6	3
B		0	4	6
C			0	7
D				0

2. **UPGMA.**

   Simulate the UPGMA algorithm with the distance matrix given below. Check that the distances given by the resulting tree correspond to the distance matrix.

	A	B	C	D	E
A	0	6	10	10	6
B		0	10	10	2
C			0	4	10
D				0	10
E					0

3. **Neighbor joining.**

   Simulate the neighbor joining method with the distance matrix given below. Check that the distances given by the resulting tree correspond to the distance matrix.

	A	B	C	D
A	0	4	4	5
B		0	6	3
C			0	7
D				0

4. **Three-point condition.**

   Consider a symmetric distance matrix $D = \{d_{ij} \mid 1 \le i \le n, 1 \le j \le n\}$. Show that the following two statements of the *three-point condition* are equivalent:

   - For all i, j, k, $d_{ij} \le \max(d_{ik}, d_{kj})$.
   - For all i, j, k, two of the values d_{ij}, d_{ik}, and d_{kj} are equal and the third one is smaller than or equal to the others.

5. **Ultrametric condition.**

   Consider a symmetric distance matrix $D = \{d_{ij} \mid 1 \le i \le n, 1 \le j \le n\}$ defined by an ultrametric tree. Show that D satisfies the three-point condition (see Problem 4 above).