58093 String Processing Algorithms (Autumn 2016)
Exercises 5 (Tuesday, November 29)

1. A don't care character # is a special character that matches any single character. For example, the pattern `#oke#i` matches `sokeri`, `pokeri` and `tokeni`.
   (a) Modify the Shift-And algorithm to handle don't care characters.
   (b) It may appear that the Morris–Pratt algorithm can handle don't care characters almost without change: Just make sure that the character comparisons are performed correctly when don't care characters are involved. However, such an algorithm would be incorrect. Give an example demonstrating this.

2. Let $\mathcal{P}_k = \{P_1, \ldots, P_{2k}\}$ be a set of patterns such that
   - for $i \in [1..k]$, $P_i = \mathtt{a}^i$ and
   - for $i \in [k+1..2k]$, $P_i = P'_i\mathtt{a}^k$ such that $|P'_i| = k$ and each P'_i is different.

   (a) Show that the total size of the sets $\mathit{patterns}(\cdot)$ in the Aho–Corasick automaton for $\mathcal{P}_k$ is asymptotically larger than $||\mathcal{P}_k||$.
   (b) Describe how to represent the sets $\mathit{patterns}(\cdot)$ so that
   - the total space complexity is never more than $\mathcal{O}(||\mathcal{P}||)$ for any $\mathcal{P}$
   - each set $\mathit{patterns}(\cdot)$ can be listed in linear time in its size.

3. Show that edit distance is a *metric*, i.e., that it satisfies the metric axioms:
   - $ed(A, B) \geq 0$
   - $ed(A, B) = 0$ if and only if $A = B$
   - $ed(A, B) = ed(B, A)$ (symmetry)
   - $ed(A, C) \leq ed(A, B) + ed(B, C)$ (triangle inequality)

4. Let $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$. Define the function $\gamma : \Sigma \times \Sigma \to \mathbb{R}_{\geq 0}$ as follows
$$\begin{aligned}
\gamma(\mathtt{a},\mathtt{a}) = \gamma(\mathtt{b},\mathtt{b}) = \gamma(\mathtt{c},\mathtt{c}) = 0\\
\gamma(\mathtt{a},\mathtt{b}) = \gamma(\mathtt{b},\mathtt{c}) = \gamma(\mathtt{c},\mathtt{a}) = 0.5\\
\gamma(\mathtt{b},\mathtt{a}) = \gamma(\mathtt{c},\mathtt{b}) = \gamma(\mathtt{a},\mathtt{c}) = 1.5
\end{aligned}$$
   Let ed_γ be a *weighted edit distance*, where the cost of substituting a character x with a character y is $\gamma(x, y)$. The cost of insertions and deletions is 1.
   (a) It might seem that we can compute $ed_\gamma(A, B)$ using the recurrence for the standard edit distance (slide 117 on the lecture notes) except δ is replaced by γ. Show that this is not the case by providing an example for which the recurrence produces an incorrect distance.
   (b) Is ed_γ a metric?

5. Let $P = \mathtt{evete}$ and $T = \mathtt{neeteneeveteen}$. Use Ukkonen's cut-off algorithm to find the occurrences of P in T for $k = 1$.