Advanced Data Structures (spring 2007)

Exercise 1 (Wed 21.3, 12-14, C221)

1. Cartesian tree.

   a) Give an example where one step of the incremental Cartesian tree construction algorithm takes linear time in the current number of nodes in the tree.
   
   b) Why does the whole algorithm still take only linear time?
   
   c) What does the term *amortized constant time* mean?

2. Range minima on a stream.

   Let $A[1, n]$ be an array of integers. *Sliding window minima* problem is to slide a window (range) of length $\alpha > 0$ through $A$ from left to right reporting the minimum value inside the sliding window at each step. Develop an algorithm that solves the sliding window minima problem in $O(n)$ time.

3. Four Russians technique.

   Develop an $O(n)$ bits structure that answers $\text{rangesum}(B, i, j)$ in constant time.

4. Lowest common ancestor queries.

   Search for articles that use LCA-queries (least/lowest/nearest common ancestor queries). Print one such article, bring it with you, and prepare to answer in few sentences what is the role of LCA-queries in the article.