Probabilistic Models: Spring 2013
Exercise Session 1 (Exercises 1-4)

Instructions: All course participants are requested to submit their exercise solutions as follows:

- Deadline: before 12 o'clock noon of the Wednesday when the corresponding exercise session will be held
- Submission as a PDF file by email to joonas.paalasmaa at helsinki.fi (cc: to petri.myllymaki at cs.helsinki.fi)
- If you write the solutions by hand, please scan your paper. However, we strongly recommend that you type your solution by using a word processor. LaTeX is of course especially suitable for typesetting math, and it also has a convenient front-end LyX.
- Use as the title of your paper: "ProMo-2013, Exercise session n, yourlastname."
- Use as the subject line of the message: "ProMo-2013, Exercise session n, yourlastname."
- In all the exercises, do not just give the answer, but also the derivation of how you obtained it.
- Participants are encouraged to write computer programs to derive solutions whenever appropriate. In this case, please enclose the program source code too as a separate file.

After the exercise session, you are allowed to send a modified version of ONE of the solutions you sent before the exercise session:

- Deadline: midnight after the exercise session.
- Submission as before, but this time please send only the modified solution, not the other (unchanged) solutions.
- Enclose the original solution first and then continue with the new material: first explain what you did wrong in the first time and then continue with modifications.
- As the title of the submission (and subject line for the email message), please use: "ProMo-2013, Exercise session n, yourlastname, modified exercise x."

Exercises 23.01.2013

1. (a) Consider two binary variables X and Y. Construct the joint probability table for the 4 probabilities $P(x, y)$ in such a way that X and Y are independent.

   (b) Continue (a) and modify the probability table so that X and Y are not independent any more.

   (c) Consider three variables X,Y and Z, and let us assume that $X \perp Y$ and $Y \perp Z$. Does it now follow that $X \perp Z$? Justify.

2. The joint probability distribution $P(X, Y)$ of two variables X and Y is given as follows:

	$X = 0$	$X = 1$	$X = 2$	$X = 3$
$Y = 0$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	0
$Y = 1$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{1}{16}$
$Y = 2$	0	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

   (a) Calculate the marginal distribution of X and Y, i.e., $P(X)$ and $P(Y)$.
   (b) Calculate $P(X|Y = 1)$.
   (c) Calculate $P(Y|X = 2)$.
   (d) Are X and Y are independent? Justitfy.

3. Continuing with the previous exercise. According to the chain rule,

$$P(X, Y) = P(X)P(Y \mid X) = P(Y)P(X \mid Y).$$

   (a) Construct two new probability tables, one for $P(X)$, and the other for $P(Y \mid X)$, so that the entries in the table for $P(X, Y)$ in the previous exercise are obtained by multiplying the corresponding entries in these two new tables together.

   (b) Construct two new probability tables, one for $P(Y)$, and the other for $P(X \mid Y)$, so that the entries in the table for $P(X, Y)$ in the previous exercise are obtained by multiplying the corresponding entries in these two new tables together.

4. (Modified from Barber, 1.18). Sally is new to the area and listens to some friends discussing about another female friend. Sally knows that they are talking about either Alice or Bella but doesn't know which. From previous conversations Sally knows some independent pieces of information: She's 80% sure that Alice has a white car, but doesn't know if Bella's car is white or black. Similarly, she's 90% sure that Bella likes sushi, but doesn't know if Alice likes sushi. Sally hears from the conversation that the person being discussed hates sushi and drives a white car. What is the probability that the friends are talking about Alice?