Probabilistic Models: Spring 2013
Exercise session 3 (exercises 10–13)

Instructions: All course participants are requested to submit their exercise solutions as follows:

- Deadline: before 12 o'clock noon of the Wednesday when the corresponding exercise session will be held
- Submission as a PDF file by email to joonas.paalasmaa at helsinki.fi (cc: to petri.myllymaki at cs.helsinki.fi)
- If you write the solutions by hand, please scan your paper. However, we strongly recommend that you type your solution by using a word processor. LaTeX is of course especially suitable for typesetting math, and it also has a convenient front-end LyX.
- Use as the title of your paper and subject of the email:
  "ProMo-2013, Exercise session n, yourlastname"
- Use as the file name:
  "ProMo-2013, Exercise session n, yourlastname.pdf"
- In all the exercises, do not just give the answer, but also the derivation of how you obtained it.
- Participants are encouraged to write computer programs to derive solutions whenever appropriate. In this case, please enclose the program source code too as a separate file.

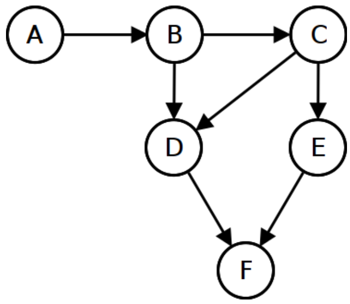
After the exercise session, you are allowed to send a modified version of one of the solutions you sent before the exercise session:

- You can only modify a solution that you submitted before the 12 o'clock deadline.
- You can only send a modified version for one solution, and only if you attended the exercise session.
- Deadline: midnight after the exercise session.
- Submission as before, but this time please send only the modified solution, not the other (unchanged) solutions. Enclose the original solution first and then continue with the new material: first explain what you did wrong in the first time and then continue with modifications.
- As the title of the submission, please use:
  "ProMo-2013, Exercise session n, yourlastname, modified exercise x."

10.

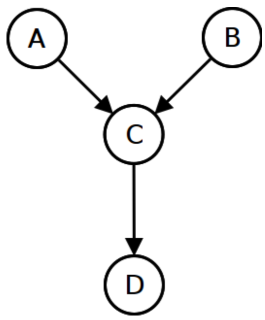
a) Show by using the d-separation criterion that a node in a Bayesian network is conditionally independent of all the other nodes, given its Markov blanket.

b) List all d-separations in the following DAG (for each pair of d-separated variables, it is enough to give one set that d-separates them):



11.

a) List all the marginal and conditional pairwise independencies represented by the following DAG:



b) Given the above Bayesian network, suppose the variables are binary and the model parameters (conditional probabilities) are as follows:

$$\begin{aligned}
P(A=1) &= 0.80 \\
P(B=1) &= 0.20 \\
P(C=1|A=0,B=0) &= 0.30 \\
P(C=1|A=1,B=0) &= 0.45 \\
P(C=1|A=0,B=1) &= 0.85 \\
P(C=1|A=1,B=1) &= 0.65 \\
P(D=1|C=1) &= 0.75 \\
P(D=1|C=0) &= 0.30
\end{aligned}$$

i) Construct the joint probability table of the distribution represented by this model.

ii) What is $\max P(A,B,C,D)$?

iii) Compute the marginal probabilities $P(A)$, $P(B)$, $P(C)$ and $P(D)$.

iv) Compute the following conditional probabilities:

$$
\begin{aligned}
P(A = 1|D = 1)\\
P(B = 1|D = 1)\\
P(A = 1|D = 0)\\
P(B = 1|D = 0)\\
P(A = 1|C = 1)\\
P(B = 1|C = 1)\\
P(A = 0|C = 1)\\
P(B = 0|C = 1)\\
P(C = 1|D = 0)\\
P(C = 1|A = 1)\\
P(D = 1|A = 1)\\
P(A = 1|C = 1, D = 1)\\
P(B = 1|C = 1, D = 1)\\
P(A = 1|C = 1, D = 0)\\
P(B = 1|C = 1, D = 0)\\
P(A = 1|C = 1, B = 1)\\
P(A = 1|C = 1, B = 0)\\
P(A = 1|B = 1)\\
P(A = 1|B = 0).
\end{aligned}
$$

12. List all 3-node Bayesian networks, and partition them into equivalence classes. For the notation, let us call the nodes X, Y and Z, and let XY means that there is a directed arc from X to Y. For example, $\{\}$ is an empty network with no arcs, and $\{XY, XZ, YZ\}$ is a fully connected network with arcs $X \to Y$, $X \to Z$ and $Y \to Z$.

13. [Friedman and Koller 3.18] An edge $X \to Y$ in graph G is said to be *covered* if $Pa_G(Y) = Pa_G(X) \cup \{X\}$. Covered edges have the nice property that for every pair of Markov equivalent networks G and G', there exists a sequence of covered edge reversal operations that converts G to G'.

a) Let G be a graph with a covered edge $X \to Y$ and G' the graph that results by reversing the edge $X \to Y$ to produce $Y \to X$, but leaving everything else unchanged. Prove that G and G' are Markov equivalent.

b) Provide a counterexample to this result in the case where $X \to Y$ is not a covered edge.