CCN Exercises session 5, to be discussed on Thu 25th Feb

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Note that no bonus points are given for these exercises (or any during the second half of the course).

1 Written exercises

1. Show that if the expectations of the grey-scale values of the pixels are the same for all x,y:

$$E\{I(x,y)\} = E\{I(x',y')\} \text{ for any } x,y,x',y' \tag{1}$$

   then removing the DC component implies than the expectation of $\tilde{I}(x,y)$ is zero for any x,y.

2. Show that if $\sum_{x,y} W_{x,y} = 0$, the removal of the DC component has no effect on the output of the features detector.

3. To get used to matrix notation:

   (a) The covariance matrix of the vector $\mathbf{x} = (x_1,\ldots,x_n)^T$ is defined as the matrix $\mathbf{C}$ with elements $c_{ij} = \text{cov}(x_i,x_j)$. Under what condition do we have $\mathbf{C} = E\{\mathbf{x}\mathbf{x}^T\}$?

   (b) Show that the covariance matrix of $\mathbf{y} = \mathbf{M}\mathbf{x}$ equals $\mathbf{M}\mathbf{C}\mathbf{M}^T$

4. Show that if the vector $\mathbf{y} = (y_1,\ldots,y_n)^T$ is white, any orthogonal transformation of that vector, $\mathbf{U}\mathbf{y}$ for an orthogonal matrix $\mathbf{U}$, is white as well.

5. Show that if $f(x)$ is a (strictly) convex function, i.e. fulfils Eq. (6.6), $f(x) + ax + b$ has the same property, for any constants a,b.

6. Show that the kurtosis of a gaussian random variable is zero. (For simplicity, assume the variable is standardized to zero mean and unit variance. Hint: try integration by parts to calculate the fourth moment.)

2 Computer assignments

Code for sampling patches from images will be provided on the main home page.

1. Take a large sample of extremely small patches of the images, so that the patch contains just two neighbouring pixels. Convert the pixels to grey-scale if necessary. Make a scatter plot `plt.scatter` of the pixels. What can you see? Compute the correlation coefficient `np.corrcoef` of the pixel values.

2. Using the same patches, convert them into two new variables: the sum of the grey-scale values and and their difference. Do the scatter plot and compute the correlation coefficient.

3. Using the same images, take a sample of 1,000 patches of the form of 1×10 pixels. Compute the covariance matrix. Plot the covariance matrix (because the patches are one-dimensional, you can easily plot this two-dimensional matrix).

4. The same as above, but remove the DC component of the patch. How does this change the covariance matrix?

5. Consider still the same 1×10 patches. Construct a simple edge detector. Compute its output when input these patches. Plot the histogram `plt.hist` of the output, and compute its kurtosis.

6. Compute the kurtoses of the original pixels in the patches. Compare with the kurtosis of the edge detector.