CCN Exercises session 6, to be discussed on Thu 3rd March

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Note that no bonus points are given for these exercises (or any during the second half of the course).

1 Written exercises

1. Prove (7.2).
2. Based on (7.2), prove that two independent random variables are uncorrelated.
3. Calculate the kurtosis of the uniform distribution in (7.23).
4. Calculate the kurtosis of the Laplacian distribution in (7.18).
5. Show that the skewness of a random variable with a pdf which is even-symmetric (i.e. $p(-x) = p(x)$) is zero.
6. In this exercise we consider a very simple case of gaussian mixtures (see Section 7.8.3). Assume that a component follows $s = vz$ where z is gaussian with zero mean and unit variance. Let us assume that in 50% of the natural images the variance coefficient v has value α. In the remaining 50% of natural images, v has value β.
   (a) What is the distribution of the random variable s in the set of all natural images? (Give the density function $p(s)$)
   (b) Show that $E\{s^2\} = \frac{1}{2}(\alpha^2 + \beta^2)$
   (c) Show that $E\{s^4\} = \frac{3}{2}(\alpha^4 + \beta^4)$
   (d) What is the kurtosis of this distribution?
   (e) Show that the kurtosis is positive for almost any parameter values.
7. Prove (7.31).

2 Computer assignments

1. Sample data (say, 1,000 data points) from the Laplacian and uniform distributions. Standardize the sample to unit variance. Compute their kurtoses.

2. Sample data from two independent Laplacian variables. Standardize both to unit variance. Compute their sum. Standardize the sum to unit variance as well. Compute the kurtosis of the standardized sum.

3. Same as 2. but with uniform distributions.

4. How do the above relate to the central limit theorem?