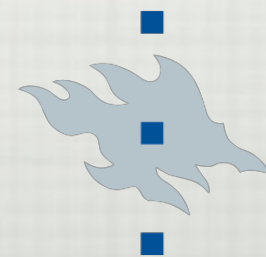




JOHDATUS TEKOÄLYYN

TEEMU ROOS



HELSINGIN YLIOPISTO

PINGVIINI(tweety) :- true.

Wulffmorgenthaler



TODENNÄKÖISYYS (TN)

EHDO LINEN TN:

$P(B \mid A)$ "B:N TODENNÄKÖISYYS, KUN TIEDE TÄÄN, ETTÄ A"
"B:N EHDOLLINEN TN ANNETTUNA A"

$$P(B \mid A) = P(A, B) / P(A), \quad \text{KUN } P(A) > 0$$

$$\Rightarrow P(A, B) = P(A) \times P(B \mid A)$$

TODENNÄKÖISYYS (TN)

1. $P(A, B) = P(A) P(B|A) = P(B) P(A|B)$ // KETJUSÄÄNTÖ

2. $P(A, B, C) = P(A, B) P(C|A, B) = P(A) P(B|A) P(C|A, B)$
// KETJUSÄÄNTÖ

3. $P(A) = P(A, B) + P(A, \neg B)$ // "MARGINALISOINTI"

4. $P(A|B) = P(A, B) / P(B)$ // EHDOLLINEN TN.

5. $A \perp B \Rightarrow P(A|B) = P(A)$ // RIIPPUMATTOMUUS

6. $A \perp B | C \Rightarrow P(A|B, C) = P(A | C)$
// EHDOLLINEN RIIPPUMATTOMUUS

7. $P(B|A) = P(B) P(A|B) / P(A)$ // BAYESIN KAAVA

TODENNÄKÖISYYS (TN)

- * SATUNNAISMUUTTUJA (SM), ESIM. NOPAN SILMÄLUKU, ON MUUTTUJA, JONKA ARVO MÄÄRÄYTYY ALKEISTAPAHTUMAN PERUSTEELLA
- * SM X ON ALKEISTAPAHTUMAN FUNKTIO $X: \Omega \rightarrow X(\Omega)$.
- * $X(\Omega)$ ON X :N ARVOJOUKKO: $X(\omega) \in X(\Omega)$ KAIKILLA $\omega \in \Omega$
- * ESIM.

KAHDEN NOPAN HEITOSSA ALKEISTAPAHTUMIEN JOUKKO ON $\Omega = \{(m,n) \mid m,n \in \{1,2,3,4,5,6\}\}$

SILMÄLUKUIEN SUMMA: $\omega = (m,n) \Rightarrow X(\omega) = m+n$
 $X(\Omega) = \{2,3,4,5,6,7,8,9,10,11,12\}$

TODENNÄKÖISYYS (TN)

MUUTTUJAN NIMET
ISOLLA

ARVOT
PIENELLÄ

- * SATUNNAISMUUTTUJEN AVULLA VOIDAAN MÄÄRITELLÄ UUSIA TAPAHTUMIA:
 $A = "X=X"$ $B = "X<4"$
 $C = "X=X, Y<5"$ $D = "X>Y"$

- * ESIM.

KAHDEN NOPAN HEITOSSA ALKEISTAPAHTUMIEN JOUKKO ON $\Omega = \{(m,n) \mid m,n \in \{1,2,3,4,5,6\}\}$

SILMÄLUKIJEN SUMMA: $\omega = (m,n) \Rightarrow X(\omega) = m+n$
 $X(\Omega) = \{2,3,4,5,6,7,8,9,10,11,12\}$

TODENNÄKÖISYYS (TN)

- * SATUNNAISMUUTTUJIEN AVULLA VOIDAAN MÄÄRITELLÄ UUSIA TAPAHTUMIA:
 $A = "X=X"$ $B = "X < 4"$
 $C = "X=X, Y < 5"$ $D = "X > Y"$

- * MUUTTUJILLA ON JAKAUMA:
 $P_X = (P(X=2), P(X=3), P(X=4), \dots, P(X=12))$
 $= (1/36, 2/36, 3/36, 4/36, 5/36, 6/36, 5/36, 4/36, 3/36, 2/36, 1/36)$

- * ESIM.

KAHDEN NOPAN HEITOSSA ALKEISTAPAHTUMIEN JOUKKO ON $\Omega = \{(m,n) \mid m,n \in \{1,2,3,4,5,6\}\}$

SILMÄLUKIJEN SUMMA: $\omega = (m,n) \Rightarrow X(\omega) = m+n$
 $X(\Omega) = \{2,3,4,5,6,7,8,9,10,11,12\}$

MUUTTUJAT VS TAPAHTUMAT

- * TAPAHTUMIEN LASKUSÄÄNNÖT, KUTEN KETJUSÄÄNTÖ, PÄTEVÄT MYÖS MUUTTUJIEN AVULLA MÄÄRITELTYIHIN TAPAHTUMIIN

- * ESIM.

$$P(X=x, Y=y) = P(X=x) \times \underline{P(Y=y \mid X=x)} \quad \forall x, y$$

- * KUSEIN LYHENNETÄÄN

$$P(x, y) = P(x) P(y \mid x) \quad \forall x, y$$

BAYES?

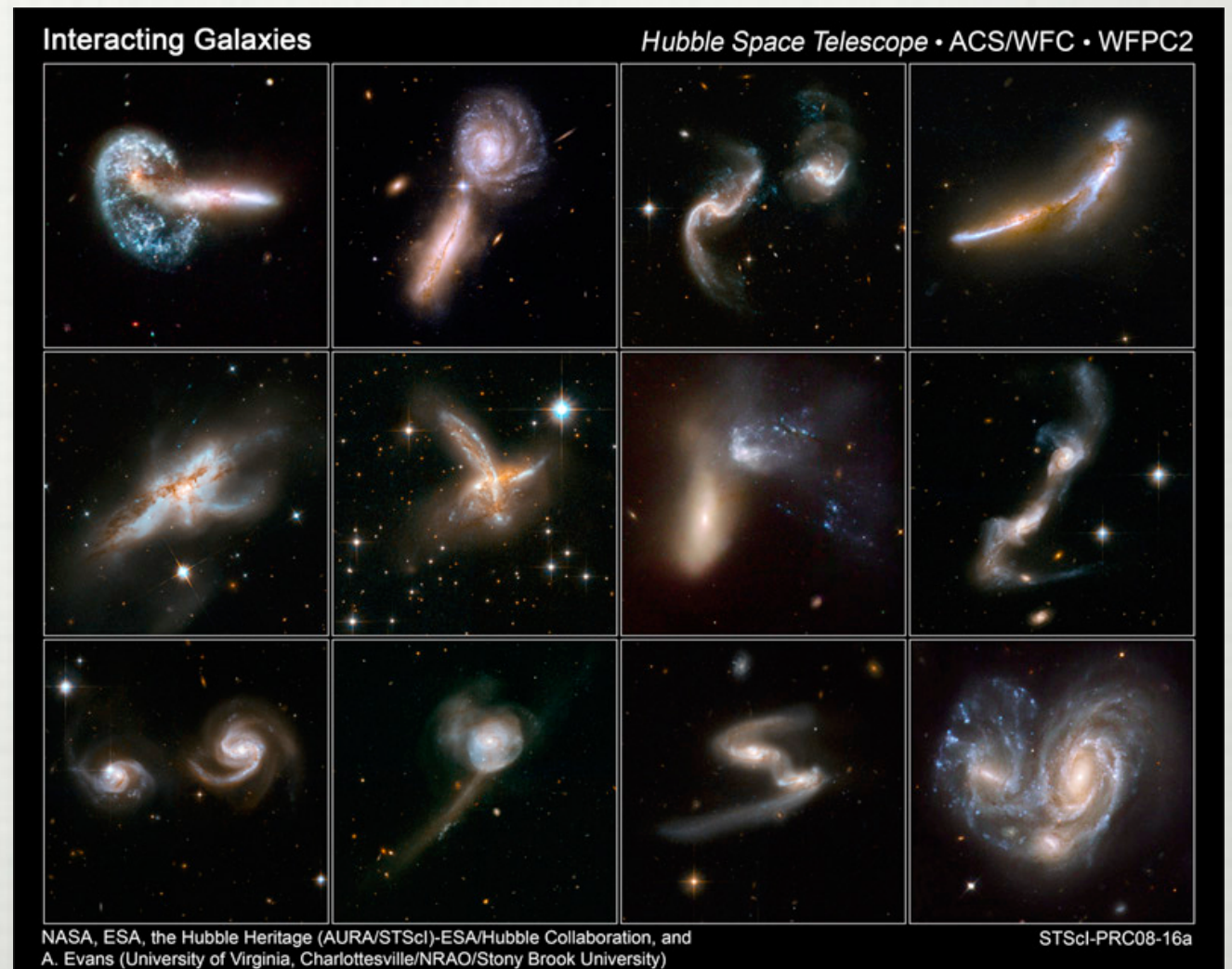
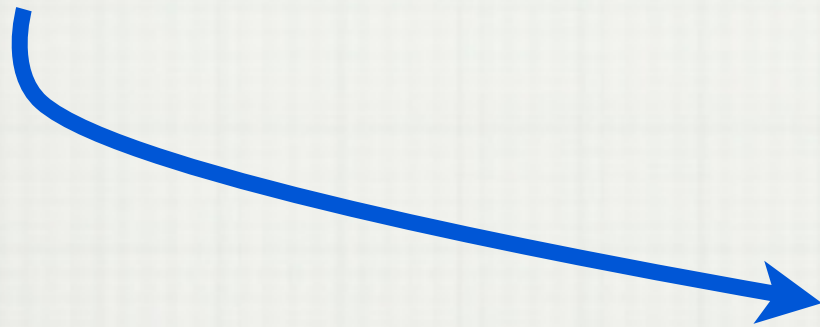


T. BAYES

BAYES?

- * $P(B|A) = P(B) P(A|B) / P(A)$

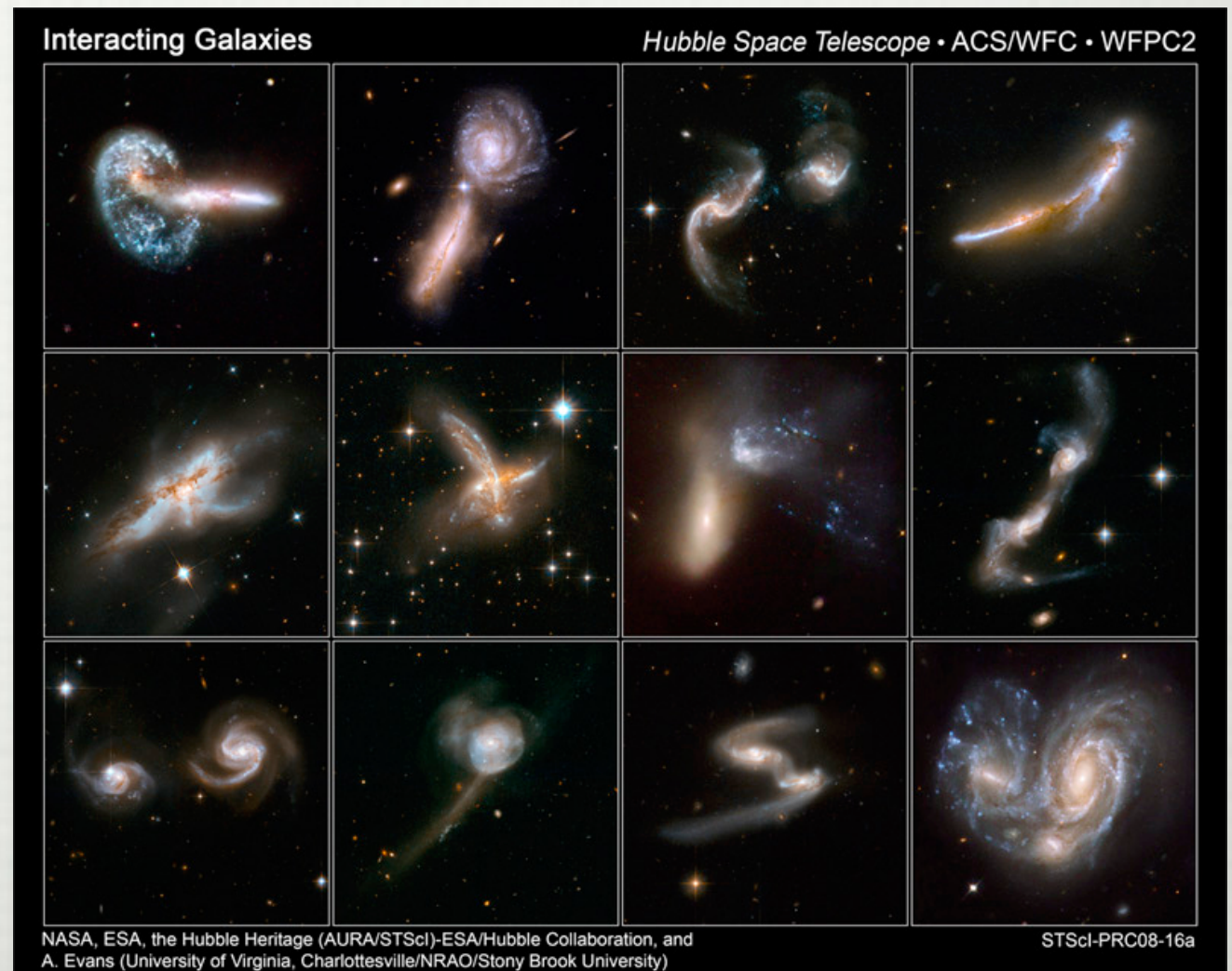
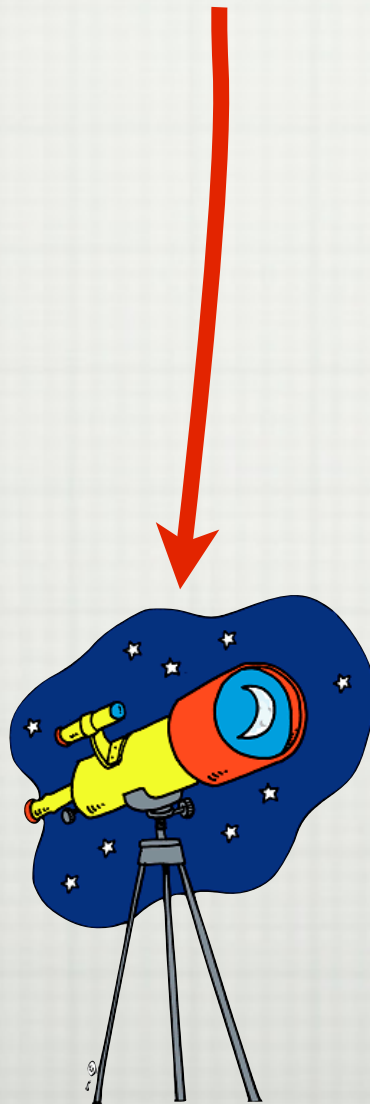
- * $P(\text{tíla} | \text{havainto}) = P(\text{tíla}) P(\text{havainto} | \text{tíla}) / P(\text{havainto})$



BAYES?

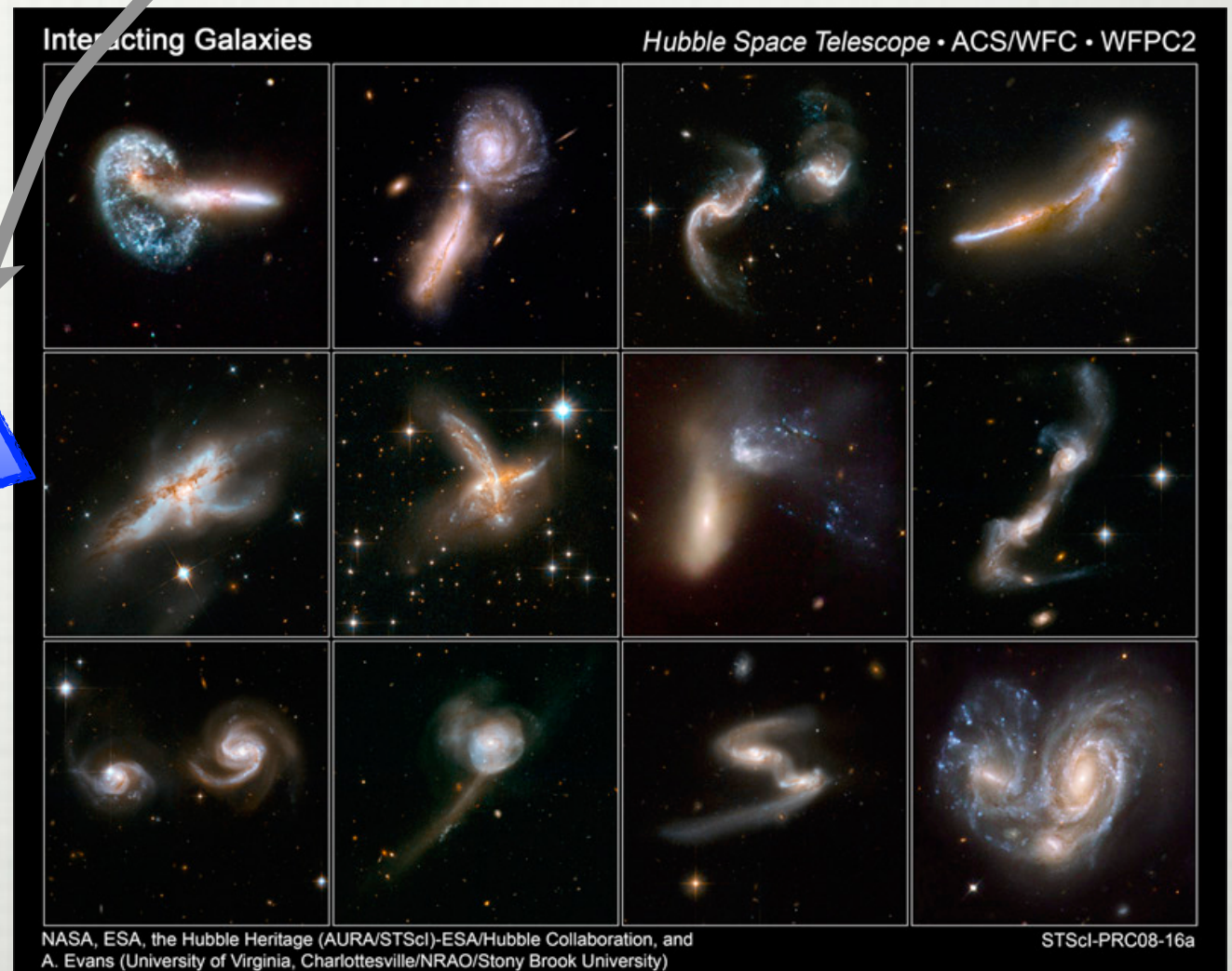
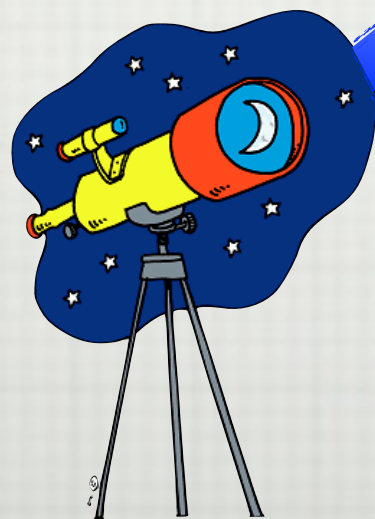
✱ $P(B|A) = P(B) P(A|B) / P(A)$

✱ $P(\text{tíla} | \text{havainto}) = P(\text{tíla}) P(\text{havainto} | \text{tíla}) / P(\text{havainto})$



BAYES?

- * $P(B|A) = P(B) P(A|B) / P(A)$
- * $P(\text{tíla} | \text{havainto}) = P(\text{tíla}) P(\text{havainto} | \text{tíla}) / P(\text{havainto})$



BAYES?

* $P(B|A) = P(B) P(A|B) / P(A)$

* $P(\text{tíla} | \text{havainto}) = P(\text{tíla}) P(\text{havainto} | \text{tíla}) / P(\text{havainto})$

↑
"POSTERIORI"

↑
"PRIORI"

↑
"USKOTTAVUUS"

↑
"ÄRSYTTÄVÄ
NIMITTÄJÄ"

BAYES?

- * $P(B|A) = P(B) P(A|B) / P(A)$
- * $P(\text{tíla} | \text{havainto}) = P(\text{tíla}) P(\text{havainto} | \text{tíla}) / P(\text{havainto})$
- * $P(\text{tíla} | \text{havainto}) = P(\text{tíla}) P(\text{havainto} | \text{tíla}) \times C$



BAYES?

- * $P(B|A) = P(B) P(A|B) / P(A)$
- * $P(\text{tíla} | \text{havainto}) = P(\text{tíla}) P(\text{havainto} | \text{tíla}) / P(\text{havainto})$
- * $P(\text{tíla} | \text{havainto}) = P(\text{tíla}) P(\text{havainto} | \text{tíla}) \times C$



BAYES?

- * $P(B|A) = P(B) P(A|B) / P(A)$
- * $P(\text{tila} | \text{havainto}) = P(\text{tila}) P(\text{havainto} | \text{tila}) / P(\text{havainto})$
- * $P(\text{tila} | \text{havainto}) = P(\text{tila}) P(\text{havainto} | \text{tila}) \times C$
- * $P(\text{tila} | h_1, h_2) = P(\text{tila} | h_1) P(h_2 | \text{tila}, h_1) \times C'$
- * $P(\text{tila} | h_1, h_2, h_3) = P(\text{tila} | h_1, h_2) P(h_3 | \text{tila}, h_1, h_2) \times C''$

ERITYISEN KÄTEVÄÄ, JOS havainnot RIIPPUMATTOMIA TOISISTAAN ANNETTUNA tila:

$$P(h_2 | \text{tila}, h_1) = P(h_2 | \text{tila}), \quad P(h_3 | \text{tila}, h_1, h_2) = P(h_3 | \text{tila}), \quad \text{jne.}$$

BAYES?

- * $P(B|A) = P(B) P(A|B) / P(A)$
- * $P(\text{tíla} | \text{havainto}) = P(\text{tíla}) P(\text{havainto} | \text{tíla}) / P(\text{havainto})$
- * $P(\text{tíla} | \text{havainto}) = P(\text{tíla}) P(\text{havainto} | \text{tíla}) \times C$
- * $P(\text{tíla} | h_1, h_2) = P(\text{tíla} | h_1) P(h_2 | \text{tíla}, \cancel{h_1}) \times C'$
- * $P(\text{tíla} | h_1, h_2, h_3) = P(\text{tíla} | h_1, h_2) P(h_3 | \text{tíla}, \cancel{h_1}, \cancel{h_2}) \times C''$
 $= P(\text{tíla}) P(h_1 | \text{tíla}) P(h_2 | \text{tíla}) P(h_3 | \text{tíla}) \times C'''$

ERITYISEN KÄTEVÄÄ, JOS havainnot RIIPPUMATTOMIA
TOISISTAAN ANNETTUNA tíla:

$$P(h_2 | \text{tíla}, h_1) = P(h_2 | \text{tíla}), \quad P(h_3 | \text{tíla}, h_1, h_2) = P(h_3 | \text{tíla}), \quad \text{jne.}$$

BAYES?

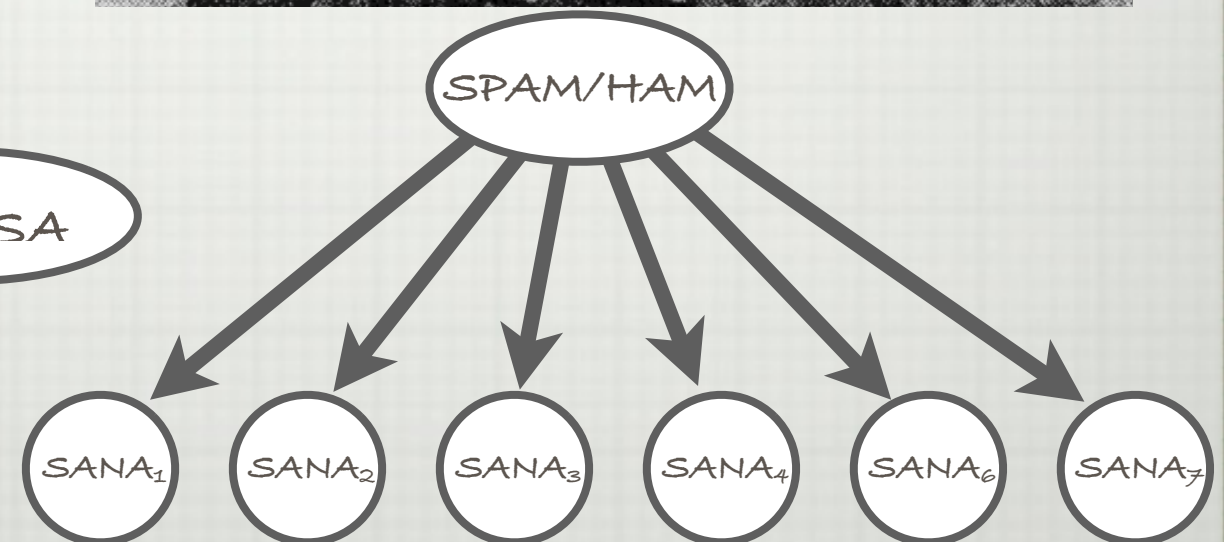
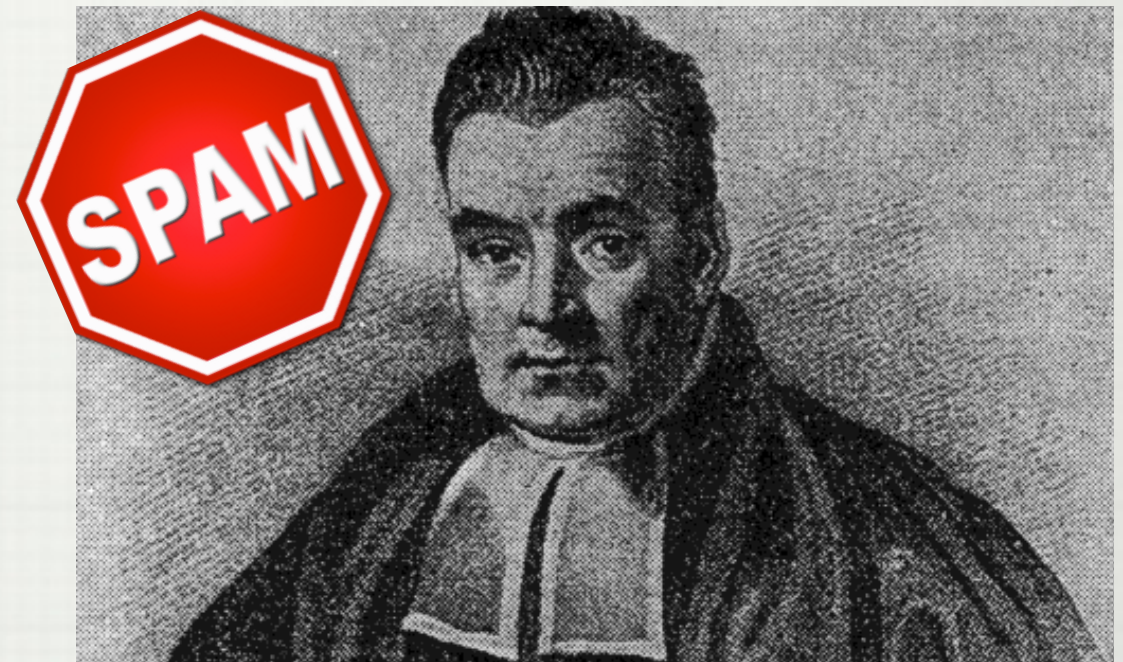
- * $P(\text{tíla} | \text{havaínto}) = P(\text{tíla}) P(\text{havaínto} | \text{tíla}) / P(\text{havaínto})$
- * $\text{Tíla} \in \{\text{saíras}, \text{terve}\}$ // potilas saíras tai terve
- * $\text{Havaínto} \in \{\text{pos}, \text{neg}\}$ // testi positiivinen tai negatiivinen
- * $P(\text{saíras}) = 0.001$ $P(\text{terve}) = \underline{0.999}$
 $P(\text{pos} | \text{saíras}) = 0.9$ $P(\text{pos} | \text{terve}) = 0.01$
 $P(\text{neg} | \text{saíras}) = \underline{0.1}$ $P(\text{neg} | \text{terve}) = \underline{0.99}$
- * $P(\text{saíras} | \text{pos}) = \underline{\sim 8 \%}$

SEURAAVAKSI

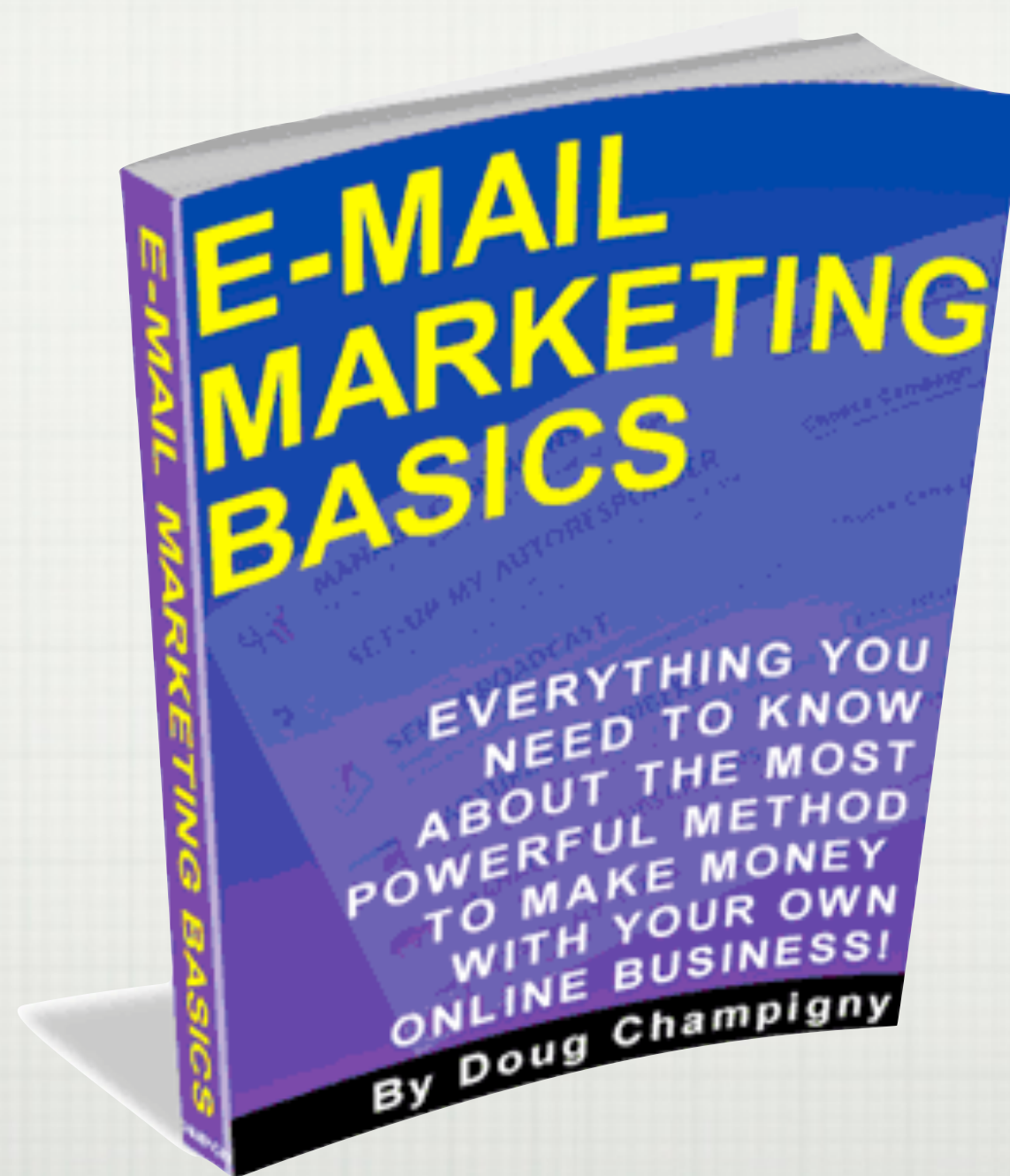
- TODENNÄKÖISYYSMALLINNUK

- SPAM-SUODATIN

- BAYES-VERKOT



KUINKA RIKASTUA

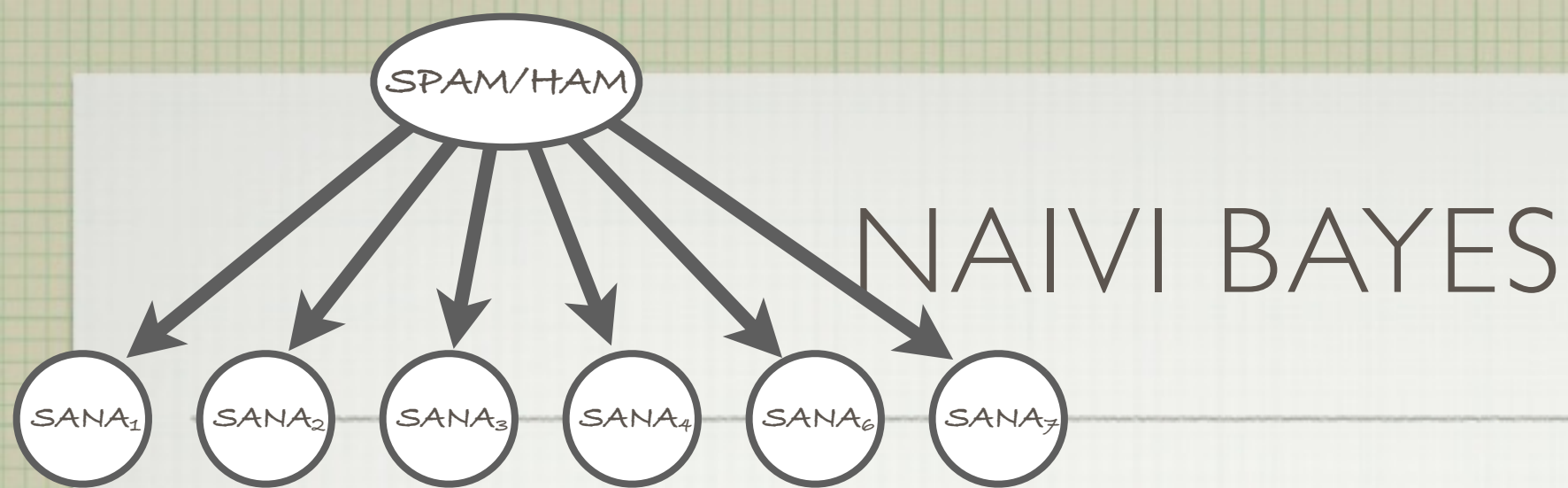


NAIVI BAYES

FROM: "MARGARETTA NITA" <MARGUERITesebrina@wmle.com>
SUBJECT: **SPECIAL OFFER : VIAGRA ON SALE AT \$1.38 !!!**
X-BOGOSITY: YES, **TESTS=BOGOFILTER, SPAMICITY=0.99993752,**
VERSION=2011-08-29
DATE: MON, 26 SEP 2011 21:52:26 +0300
X-CLASSIFICATION: **JUNK - AD HOC SPAM DETECTED (CODE = 73)**

SPECIAL OFFER : VIAGRA ON SALE AT \$1.38 !!!

COMPARE THE BEST ONLINE PHARMACIES TO **BUY VIAGRA**. ORDER
VIAGRA ONLINE WITH HUGE **DISCOUNT**.
MULTIPLE BENEFITS INCLUDE **FREE** SHIPPING, REORDER **DISCOUNTS**,
BONUS PILLS
[HTTP://RXPHARMACYCVS.RU](http://rxpharmacy CVS.RU)



ROSKAPOSTISUODATIN:

SPAMICITY(Viesti, P):

Odds = $P_{\text{Spam}} / P_{\text{noSpam}}$

for each Sana in Viesti

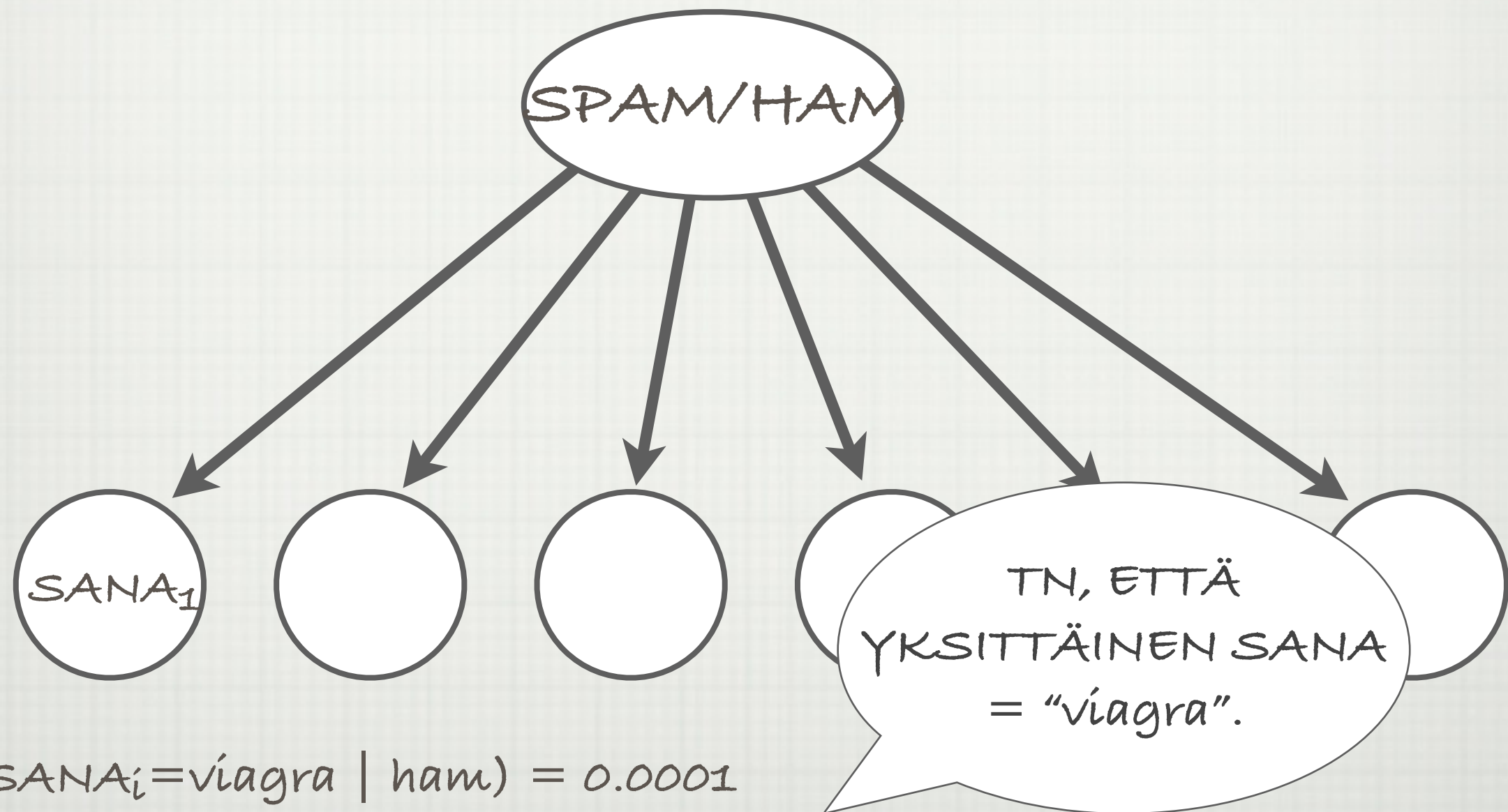
Odds = Odds * $P_{\text{Sana_Spam}}(\text{Sana}) / P_{\text{Sana_noSpam}}(\text{Sana})$

return(Odds)

JOS SPAMICITY(Viesti, P) > 1, LUOKITTELE VIEСТИ SPAMIKSI

JOS SPAMICITY(Viesti, P) < 1, LUOKITTELE VIEСТИ HAMIKSI

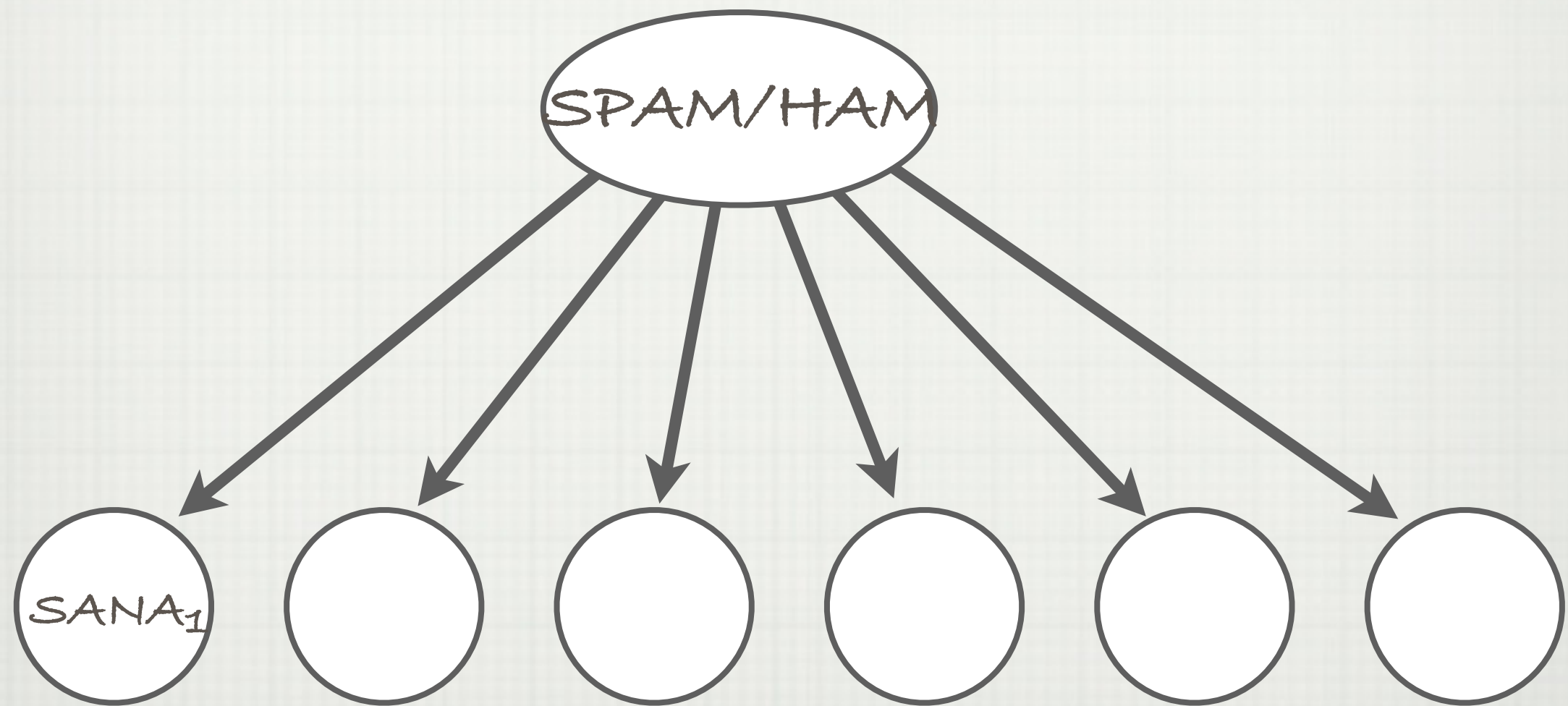
NAIVI BAYES



$$P(SANA_i = víagra \mid ham) = 0.0001$$

$$P(SANA_i = víagra \mid spam) = 0.002$$

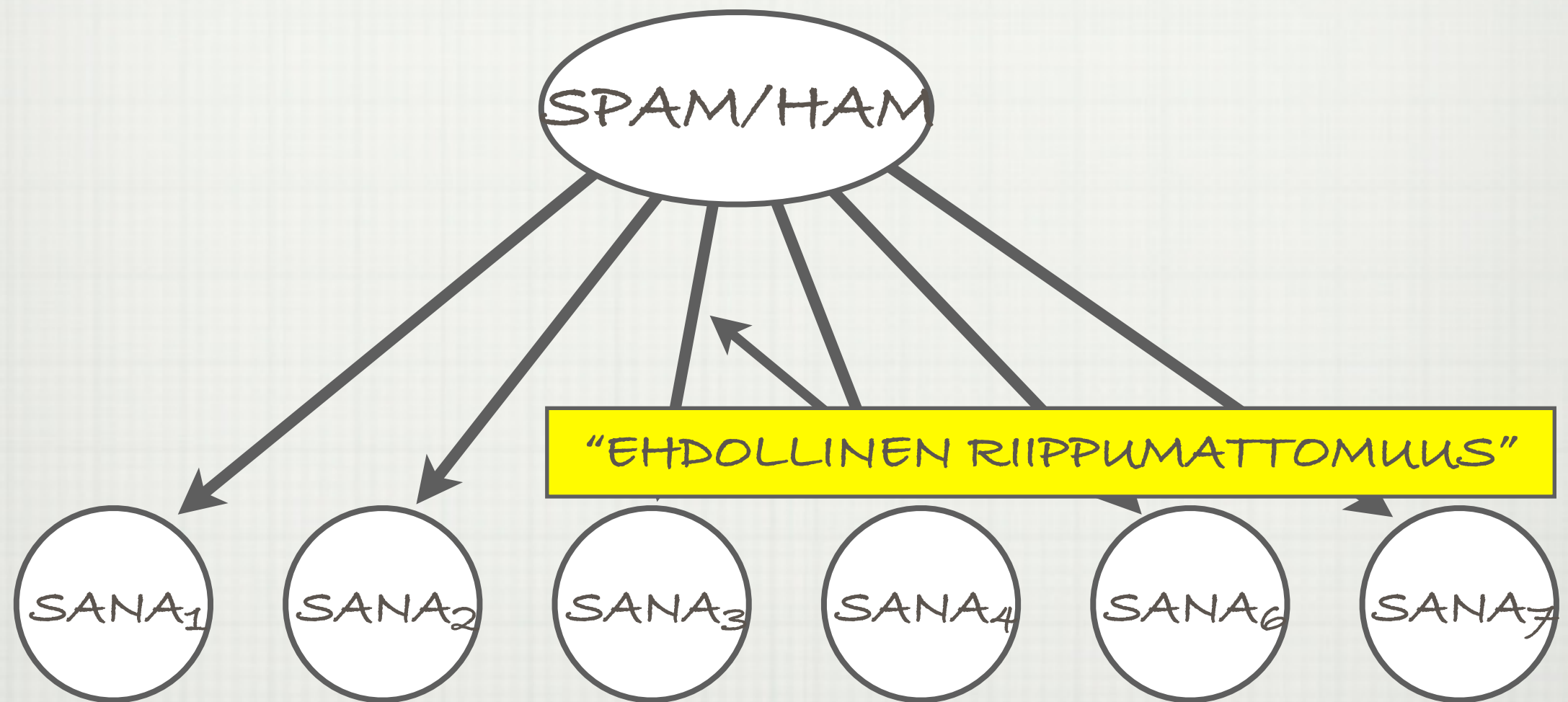
NAIVI BAYES



$$P(SANA_i = \$ \mid \text{ham}) = 0.0002$$

$$P(SANA_i = \$ \mid \text{spam}) = 0.005$$

NAIVI BAYES



NAIVI BAYES

MUUTTUJAT:

1. LUOKKA: spam/ham
2. $SANA_1$
3. $SANA_2$
4. ...

JAKAUMAT:

$$P(LUOKKA=spam) = 0.5$$

$$P(SANA_i=viagra | spam) = 0.002$$

$$P(SANA_i=viagra | ham) = 0.0001$$

$$P(SANA_i=\$ | spam) = 0.005$$

$$P(SANA_i=\$ | ham) = 0.0002$$

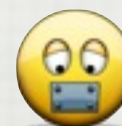
$$P(SANA_i=is | spam) = 0.002$$

$$P(SANA_i=is | ham) = 0.002$$

$$P(SANA_i=algorithm | spam) = 0.0001$$

$$P(SANA_i=algorithm | ham) = 0.002$$

JNE...



NAIVI BAYES

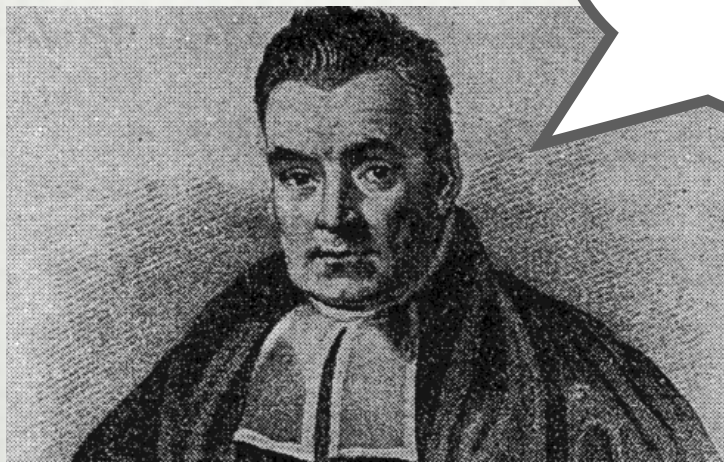
PÄÄTTELY:

1. $P(\text{spam}) = 0.5$

$P(\text{spam}) P(\text{SANA}_1 = \text{víagra} \mid \text{spam})$

2. $P(\text{spam} \mid \text{SANA}_1 = \text{víagra}) = \frac{\text{-----}}{P(\text{SANA}_1 = \text{víagra})}$

BAYESIN
KAAVA!



NAIVI BAYES

PÄÄTTELY:

1. $P(\text{spam}) = 0.5$

$$P(\text{spam}) P(\text{SANA}_1 = \text{víagra} \mid \text{spam})$$

2. $P(\text{spam} \mid \text{SANA}_1 = \text{víagra}) = \frac{\text{-----}}{P(\text{SANA}_1 = \text{víagra})}$

$$P(\text{SANA}_1 = \text{víagra}) = P(\text{spam}) P(\text{SANA}_1 = \text{víagra} \mid \text{spam}) \\ + P(\text{ham}) P(\text{SANA}_1 = \text{víagra} \mid \text{ham})$$

NAIVI BAYES

PÄÄTTELY:

1. $P(\text{spam}) = 0.5$

$$P(\text{spam}) P(\text{SANA}_1 = \text{víagra} \mid \text{spam})$$

2. $P(\text{spam} \mid \text{SANA}_1 = \text{víagra}) = \frac{P(\text{spam}) P(\text{SANA}_1 = \text{víagra} \mid \text{spam})}{P(\text{SANA}_1 = \text{víagra})}$

3. $P(\text{spam} \mid \text{SANA}_1 = \text{víagra}, \text{SANA}_2 = \text{ís})$

$$P(\text{spam}) P(\text{SANA}_1 = \text{víagra}, \text{SANA}_2 = \text{ís} \mid \text{spam})$$

$$= \frac{P(\text{spam}) P(\text{SANA}_1 = \text{víagra}, \text{SANA}_2 = \text{ís} \mid \text{spam})}{P(\text{SANA}_1 = \text{víagra}, \text{SANA}_2 = \text{ís})}$$

4. $P(\text{spam} \mid \text{SANA}_1 = \text{víagra}, \text{SANA}_2 = \text{ís}, \text{SANA}_3 = \text{algoríthm})$

$$P(\text{spam}) P(\text{SANA}_1 = \text{víagra}, \text{SANA}_2 = \text{ís}, \text{SANA}_3 = \text{algoríthm} \mid \text{spam})$$

$$= \frac{P(\text{spam}) P(\text{SANA}_1 = \text{víagra}, \text{SANA}_2 = \text{ís}, \text{SANA}_3 = \text{algoríthm} \mid \text{spam})}{P(\text{SANA}_1 = \text{víagra}, \text{SANA}_2 = \text{ís}, \text{SANA}_3 = \text{algoríthm})}$$

NAIVI BAYES

PÄÄTTELY:

1. $P(\text{spam}) = 0.5$

$$P(\text{spam}) P(\text{SANA}_1 = \text{víagra} \mid \text{spam})$$

2. $P(\text{spam} \mid \text{SANA}_1 = \text{víagra}) = \frac{P(\text{spam}) P(\text{SANA}_1 = \text{víagra} \mid \text{spam})}{P(\text{SANA}_1 = \text{víagra})}$

3. $P(\text{spam} \mid \text{SANA}_1 = \text{víagra}, \text{SANA}_2 = \text{ís})$

$$P(\text{spam}) P(\text{SANA}_1 = \text{víagra}, \text{SANA}_2 = \text{ís} \mid \text{spam})$$

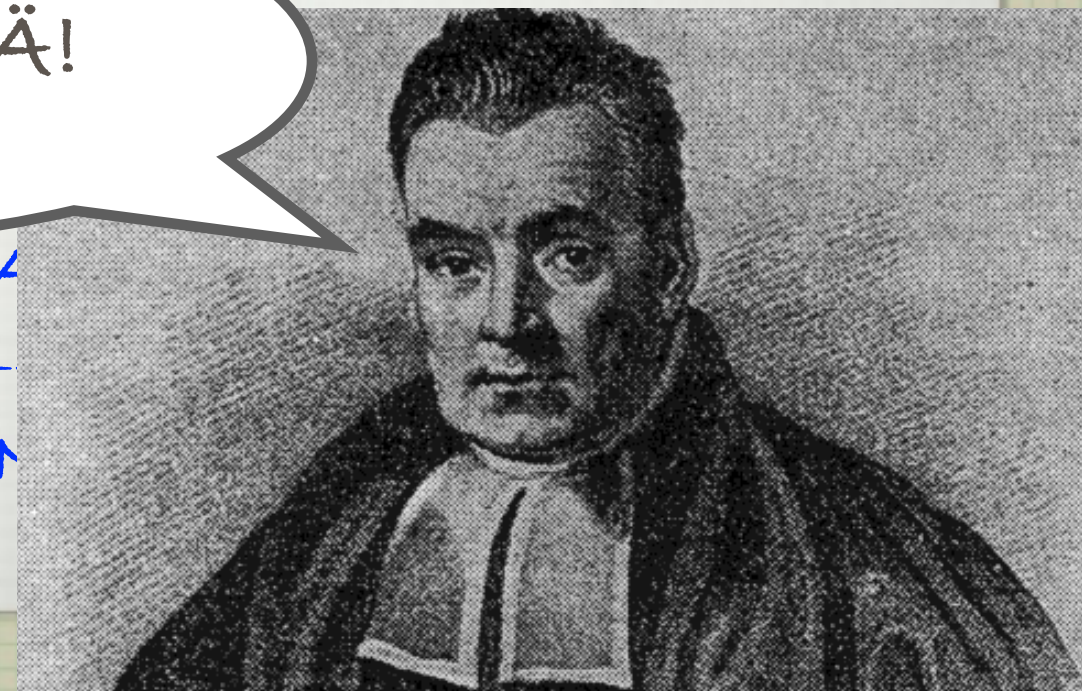
$$= \frac{P(\text{spam}) P(\text{SANA}_1 = \text{víagra} \mid \text{spam}) P(\text{SANA}_2 = \text{ís} \mid \text{spam})}{P(\text{SANA}_1 = \text{víagra}, \text{SANA}_2 = \text{ís})}$$

KAKSI NIKSIÄ!

4. $P(\text{spam} \mid \text{SANA}_1 = \text{víagra}, \text{SANA}_2 = \text{ís}, \text{SANA}_3 = \text{pöytä})$

$$P(\text{spam}) P(\text{SANA}_1 = \text{víagra} \mid \text{spam}) P(\text{SANA}_2 = \text{ís} \mid \text{spam}) P(\text{SANA}_3 = \text{pöytä} \mid \text{spam})$$

$$= \frac{P(\text{spam}) P(\text{SANA}_1 = \text{víagra} \mid \text{spam}) P(\text{SANA}_2 = \text{ís} \mid \text{spam}) P(\text{SANA}_3 = \text{pöytä} \mid \text{spam})}{P(\text{SANA}_1 = \text{víagra}, \text{SANA}_2 = \text{ís}, \text{SANA}_3 = \text{pöytä})}$$



NAIVI BAYES

#1



PÄÄTTELY:

$$4. P(\text{spam} \mid SANA_1=\text{víagra}, SANA_2=\text{ís}, SANA_3=\text{algorithm}) \\ P(\text{spam}) P(SANA_1=\text{víagra}, SANA_2=\text{ís}, SANA_3=\text{algorithm} \mid \text{spam}) \\ =$$

$$P(SANA_1=\text{víagra}, SANA_2=\text{ís}, SANA_3=\text{algorithm})$$

$$P(SANA_1=\text{víagra}, SANA_2=\text{ís}, SANA_3=\text{algorithm}) \\ = P(\text{spam}, SANA_1=\text{víagra}, SANA_2=\text{ís}, SANA_3=\text{algorithm}) \\ + P(\text{ham}, SANA_1=\text{víagra}, SANA_2=\text{ís}, SANA_3=\text{algorithm})$$

NAIVI BAYES

#1



PÄÄTTELY:

$$\begin{aligned} 4. & P(\text{spam} \mid SANA_1=\text{víagra}, SANA_2=\text{ís}, SANA_3=\text{algoríthm}) \\ & P(\text{spam}) P(SANA_1=\text{víagra}, SANA_2=\text{ís}, SANA_3=\text{algoríthm} \mid \text{spam}) \\ = & \text{-----} \\ & P(SANA_1=\text{víagra}, SANA_2=\text{ís}, SANA_3=\text{algoríthm}) \end{aligned}$$

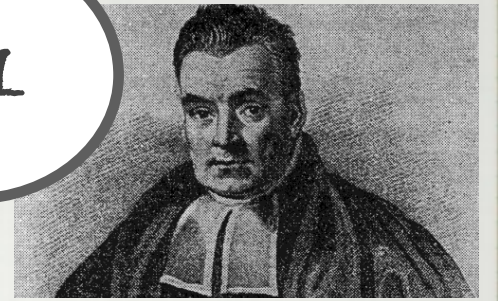
$P(\text{evidenssi})$

$$\begin{aligned} &= P(\text{spam}, \text{evidenssi}) \\ &+ P(\text{ham}, \text{evidenssi}) \end{aligned}$$

// MARGINALISOINTI

NAIVI BAYES

#1



PÄÄTTELY:

$$\begin{aligned} 4. & P(\text{spam} \mid \text{SANA}_1=\text{víagra}, \text{SANA}_2=\text{ís}, \text{SANA}_3=\text{algorithm}) \\ & P(\text{spam}) P(\text{SANA}_1=\text{víagra}, \text{SANA}_2=\text{ís}, \text{SANA}_3=\text{algorithm} \mid \text{spam}) \\ = & \text{-----} \\ & P(\text{SANA}_1=\text{víagra}, \text{SANA}_2=\text{ís}, \text{SANA}_3=\text{algorithm}) \end{aligned}$$

$P(\text{evidenssi})$

$$\begin{aligned} &= P(\text{spam}, \text{evidenssi}) \\ &+ P(\text{ham}, \text{evidenssi}) \end{aligned}$$

// MARGINALISOINTI

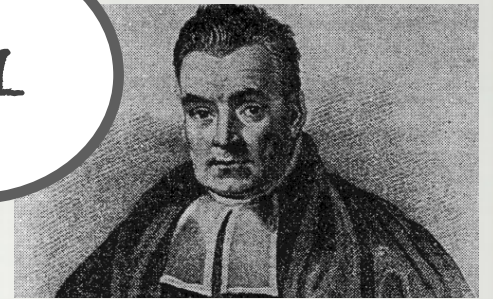
$$P(\text{spam})P(\text{evidenssi} \mid \text{spam})$$

$$\begin{aligned} P(\text{spam} \mid \text{evidenssi}) &= \text{-----} \\ & \frac{P(\text{spam}, \text{evidenssi}) + P(\text{ham}, \text{evidenssi})}{P(\text{ham})P(\text{evidenssi} \mid \text{ham})} \end{aligned}$$

$$\begin{aligned} P(\text{ham} \mid \text{evidenssi}) &= \text{-----} \\ & \frac{P(\text{spam}, \text{evidenssi}) + P(\text{ham}, \text{evidenssi})}{P(\text{spam}, \text{evidenssi}) + P(\text{ham}, \text{evidenssi})} \end{aligned}$$

NAIVI BAYES

#1



PÄÄTTELY:

$$\begin{aligned} 4. & P(\text{spam} \mid \text{SANA}_1=\text{víagra}, \text{SANA}_2=\text{ís}, \text{SANA}_3=\text{algorithm}) \\ & P(\text{spam}) P(\text{SANA}_1=\text{víagra}, \text{SANA}_2=\text{ís}, \text{SANA}_3=\text{algorithm} \mid \text{spam}) \\ = & \text{-----} \\ & P(\text{SANA}_1=\text{víagra}, \text{SANA}_2=\text{ís}, \text{SANA}_3=\text{algorithm}) \end{aligned}$$

$P(\text{evidenssi})$

$$\begin{aligned} &= P(\text{spam}, \text{evidenssi}) \\ &+ P(\text{ham}, \text{evidenssi}) \end{aligned}$$

// MARGINALISOINTI

$$P(\text{spam})P(\text{evidenssi} \mid \text{spam})$$

$$\begin{aligned} P(\text{spam} \mid \text{evidenssi}) &= \text{-----} \\ & \frac{P(\text{spam}, \text{evidenssi}) + P(\text{ham}, \text{evidenssi})}{P(\text{ham})P(\text{evidenssi} \mid \text{ham})} \end{aligned}$$

$$\begin{aligned} P(\text{ham} \mid \text{evidenssi}) &= \text{-----} \\ & \frac{P(\text{spam}, \text{evidenssi}) + P(\text{ham}, \text{evidenssi})}{P(\text{spam})P(\text{evidenssi} \mid \text{spam})} \end{aligned}$$

NAIVI BAYES

#1



PÄÄTTELY:

$$\begin{aligned} 4. & P(\text{spam} \mid \text{SANA}_1=\text{víagra}, \text{SANA}_2=\text{ís}, \text{SANA}_3=\text{algoríthm}) \\ & P(\text{spam}) P(\text{SANA}_1=\text{víagra}, \text{SANA}_2=\text{ís}, \text{SANA}_3=\text{algoríthm} \mid \text{spam}) \\ = & \text{-----} \\ & P(\text{SANA}_1=\text{víagra}, \text{SANA}_2=\text{ís}, \text{SANA}_3=\text{algoríthm}) \end{aligned}$$

$P(\text{evidenssi})$

$$\begin{aligned} &= P(\text{spam}, \text{evidenssi}) \\ &+ P(\text{ham}, \text{evidenssi}) \end{aligned}$$

// MARGINALISOINTI

$P(\text{spam} \mid \text{evidenssi})$

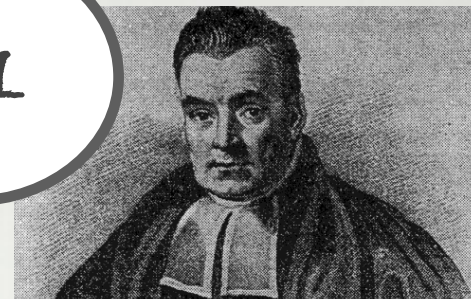
$P(\text{spam})P(\text{evidenssi} \mid \text{spam})$

$P(\text{ham} \mid \text{evidenssi})$

$P(\text{ham})P(\text{evidenssi} \mid \text{ham})$

NAIVI BAYES

#1



PÄÄTTELY:

$$4. P(\text{spam} \mid \text{SANA}_1=\text{víagra}, \text{SANA}_2=\text{ís}, \text{SANA}_3=\text{algorithm})$$
$$P(\text{spam}) P(\text{SANA}_1=\text{víagra}, \text{SANA}_2=\text{ís}, \text{SANA}_3=\text{algorithm} \mid \text{spam})$$
$$=$$

$$P(\text{SANA}_1=\text{víagra}, \text{SANA}_2=\text{ís}, \text{SANA}_3=\text{algorithm})$$

$$P(\text{evidenssi})$$
$$= P(\text{spam}, \text{evidenssi})$$
$$+ P(\text{ham}, \text{evidenssi})$$

$$P(\text{spam} \mid \text{evidenssi})$$

$$P(\text{ham} \mid \text{evidenssi})$$

// MARK

$$P(S \mid E)$$

$$P(S)P(E \mid S)$$

$$P(\neg S \mid E)$$

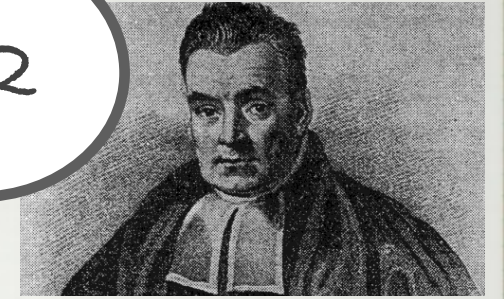
$$P(\neg S)P(E \mid \neg S)$$

$$P(\text{spam})P(\text{evidenssi} \mid \text{spam})$$

$$P(\text{ham})P(\text{evidenssi} \mid \text{ham})$$

NAIVI BAYES

#2



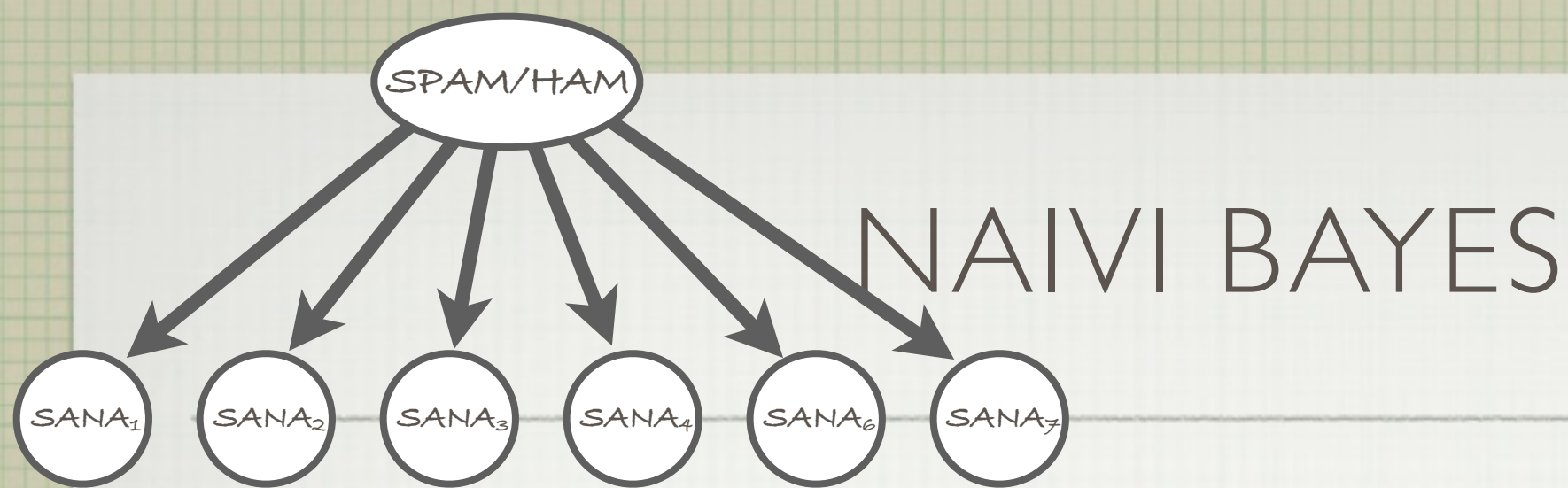
PÄÄTTELY:

$$P(SANA_1=viagra, SANA_2=is, SANA_3=algorithm | spam)$$

$$= P(SANA_1=viagra | spam) \quad // \text{KETJUSÄÄNTÖ}$$

$$P(SANA_2=is | SANA_1=viagra, spam)$$

$$P(SANA_3=algorithm | SANA_1=viagra, SANA_2=is, spam)$$



#2



PÄÄTTELY:

$$P(SANA_1=viagra, SANA_2=is, SANA_3=algorithm | spam)$$

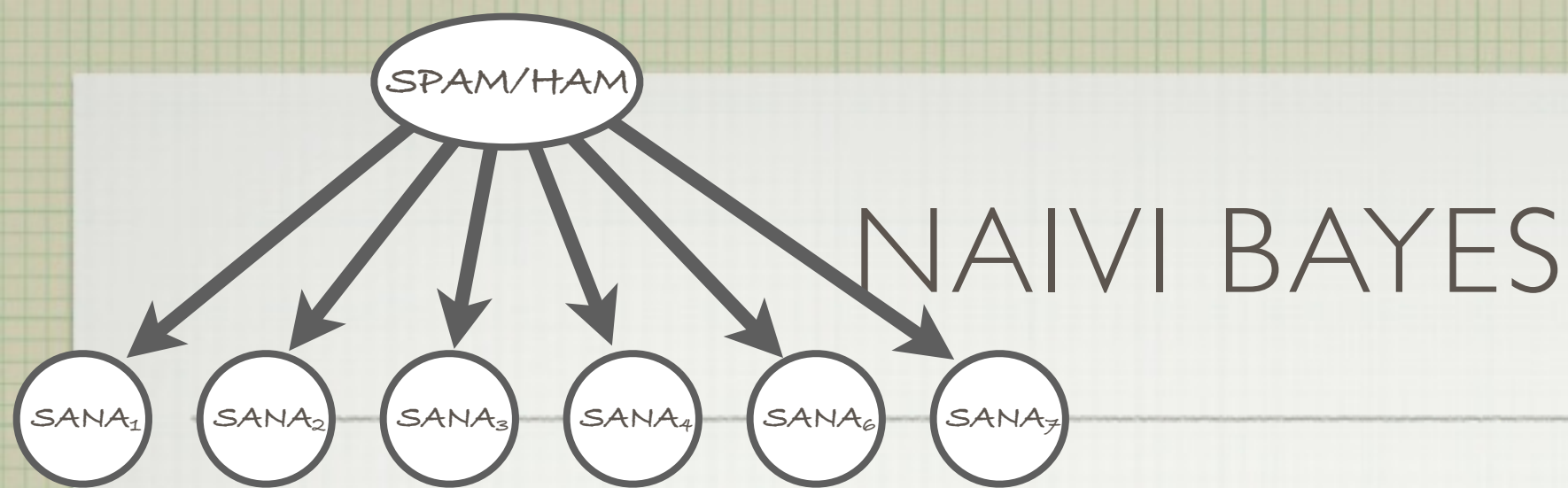
$$= P(SANA_1=viagra | spam)$$

// KETJUSÄÄNTÖ

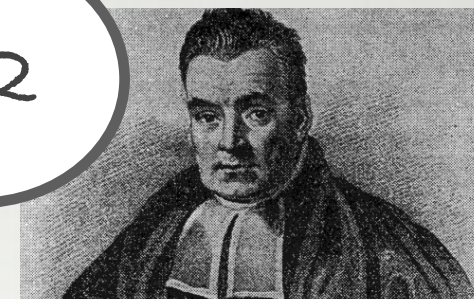
$$P(SANA_2=is | \cancel{SANA_1=viagra}, spam)$$

$$P(SANA_3=algorithm | \cancel{SANA_1=viagra}, \cancel{SANA_2=is}, spam)$$

// RIIPPUMATTOMUUS



#2



PÄÄTTELY:

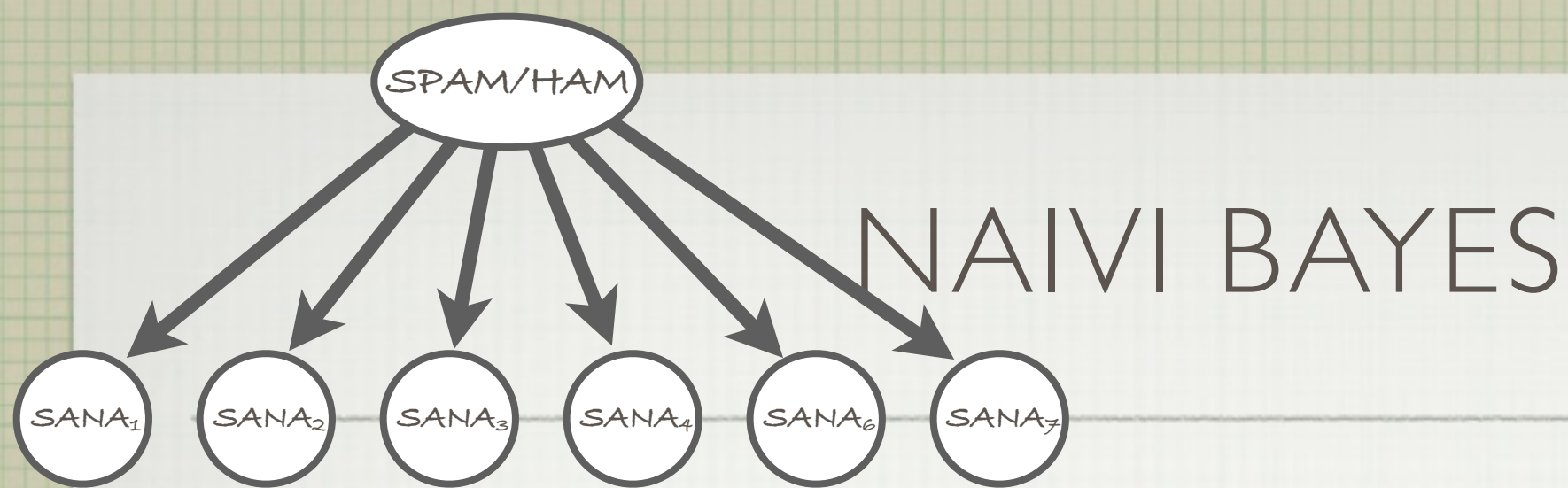
$$P(SANA_1=viagra, SANA_2=is, SANA_3=algorithm | spam)$$

$$= P(SANA_1=viagra | spam)$$

$$P(SANA_2=is | spam)$$

$$P(SANA_3=algorithm | spam)$$

// KETJUSÄÄNTÖ



#2



PÄÄTTELY:

$$P(SANA_1=viagra, SANA_2=is, SANA_3=algorithm|spam)$$

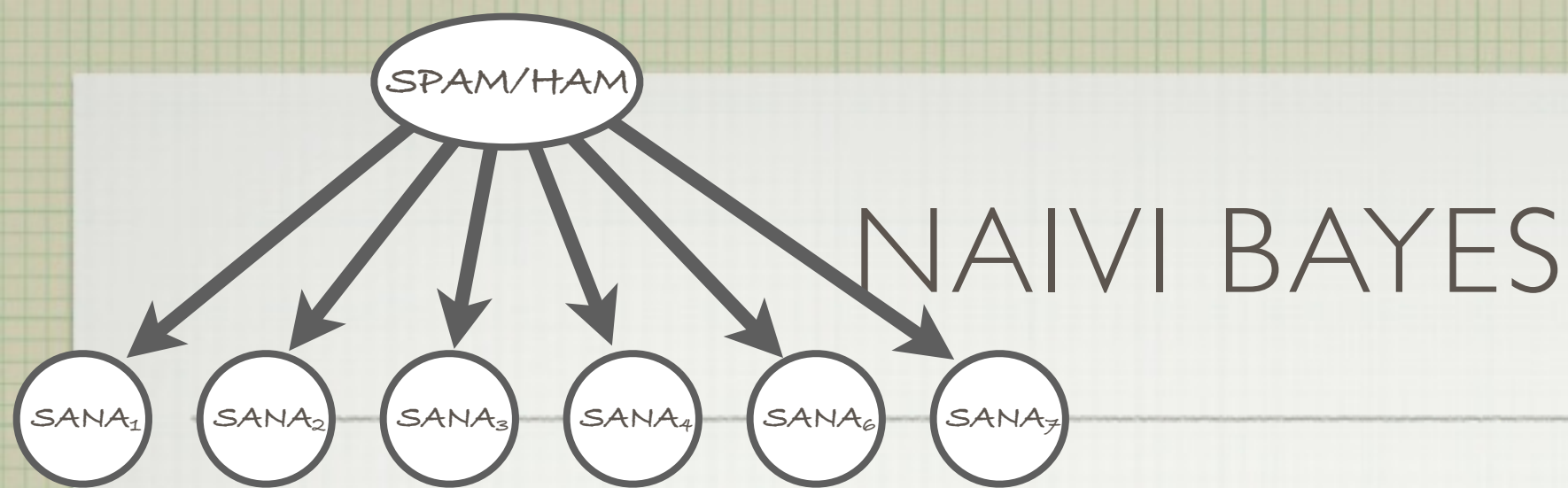
$$= P(SANA_1=viagra|spam) \\ P(SANA_2=is|spam) \\ P(SANA_3=algorithm|spam)$$

$$\frac{P(S|E) \quad P(S)P(E|S)}{P(\neg S|E) \quad P(\neg S)P(E|\neg S)}$$

$$P(SANA_1=viagra, SANA_2=is, SANA_3=algorithm|ham)$$

$$= P(SANA_1=viagra|ham) \\ P(SANA_2=is|ham) \\ P(SANA_3=algorithm|ham)$$

// KETJUSÄÄNTÖ



#2



PÄÄTTELY:

$$P(\text{spam} | \text{evidenssi}) / P(\text{ham} | \text{evidenssi})$$

$$= P(\text{spam}) / P(\text{ham})$$

$$P(\text{SANA}_1 = \text{viagra} | \text{spam}) / P(\text{SANA}_1 = \text{viagra} | \text{ham})$$

> 1

$$P(\text{SANA}_2 = \text{is} | \text{spam}) / P(\text{SANA}_2 = \text{is} | \text{ham})$$

= 1

$$P(\text{SANA}_3 = \text{algorithm} | \text{spam}) / P(\text{SANA}_3 = \text{algorithm} | \text{ham})$$

< 1

...

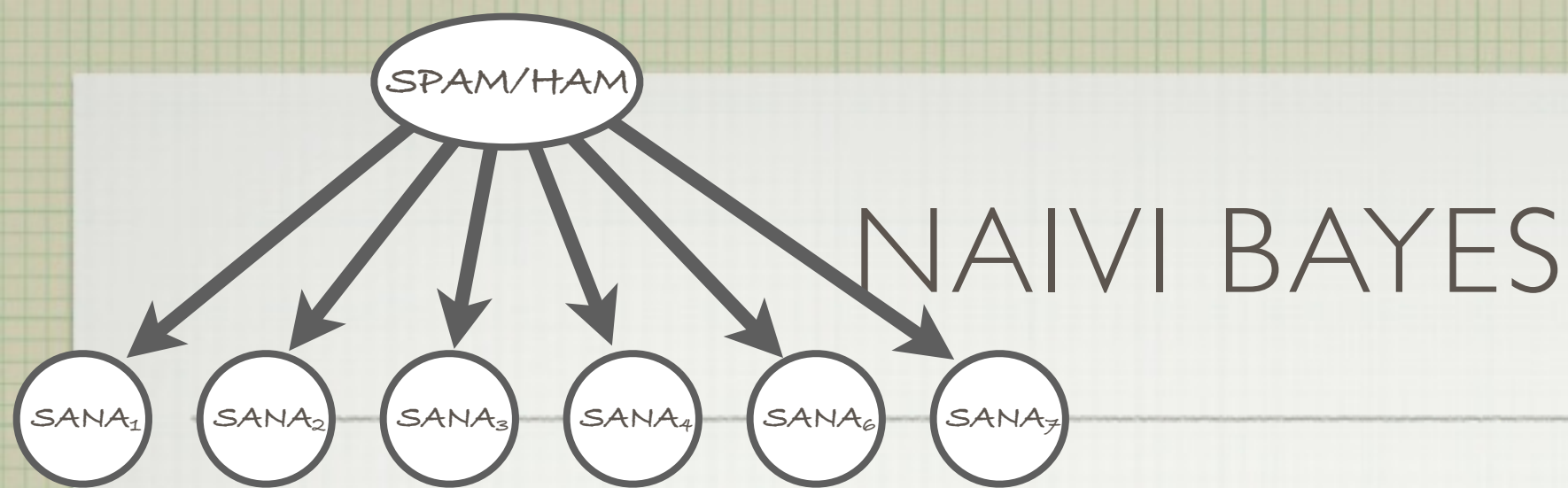
...

...

OSAMÄÄRÄ!

> 1 => LUOKITTELE: "SPAM"

< 1 => LUOKITTELE: "HAM"



PSEUDOKOODINA:

SPAMICITY(Viesti, P):

Odds = P.Spam / P.noSpam

for each Sana in Viesti

Odds = Odds * P.Sana_Spam(Sana) / P.Sana_noSpam(Sana)

return(Odds)

PÄÄTTELY:

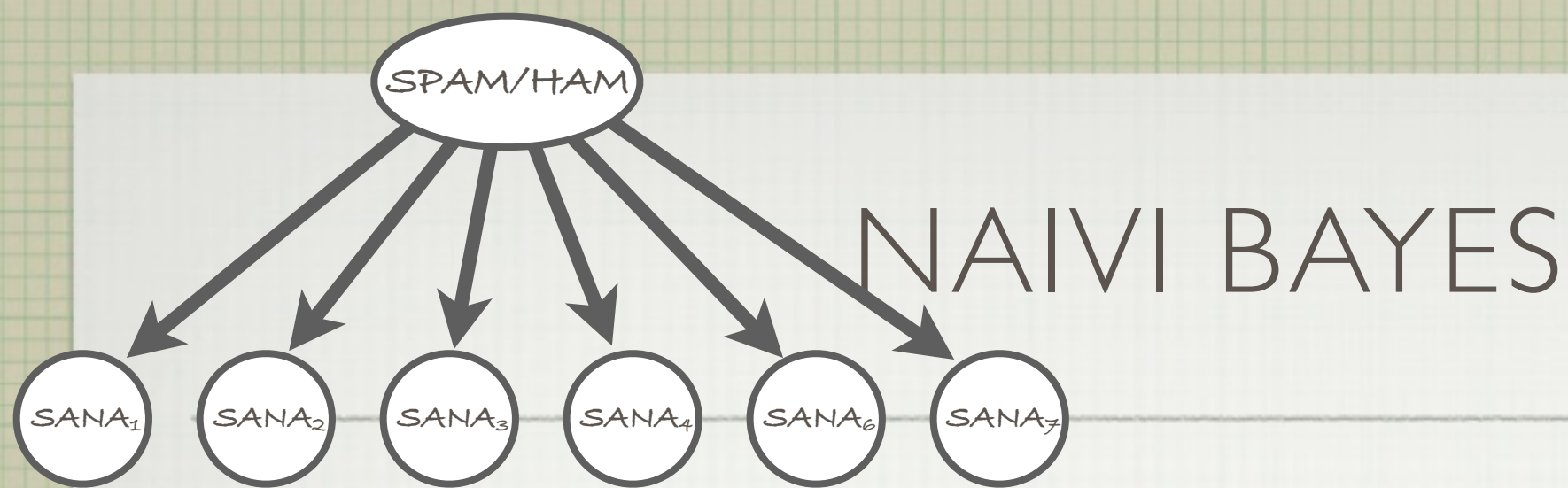
$$P(\text{spam} | \text{EVIDENSSI}) / P(\text{ham} | \text{EVIDENSSI})$$

$$= P(\text{spam}) / P(\text{ham})$$

$$P(\text{SANA}_1 = \text{viagra} | \text{spam}) / P(\text{SANA}_1 = \text{viagra} | \text{ham})$$

$$P(\text{SANA}_2 = \text{is} | \text{spam}) / P(\text{SANA}_2 = \text{is} | \text{ham})$$

$$P(\text{SANA}_3 = \text{algorithm} | \text{spam}) / P(\text{SANA}_3 = \text{algorithm} | \text{ham})$$



PSEUDOKOODINA:

SPAMICITY(Viesti, P):

Odds = P.Spam / P.noSpam

for each Sana in Viesti

Odds = Odds * P.Sana_Spam(Sana) / P.Sana_noSpam(Sana)

return(Odds)

PÄÄTTELY:

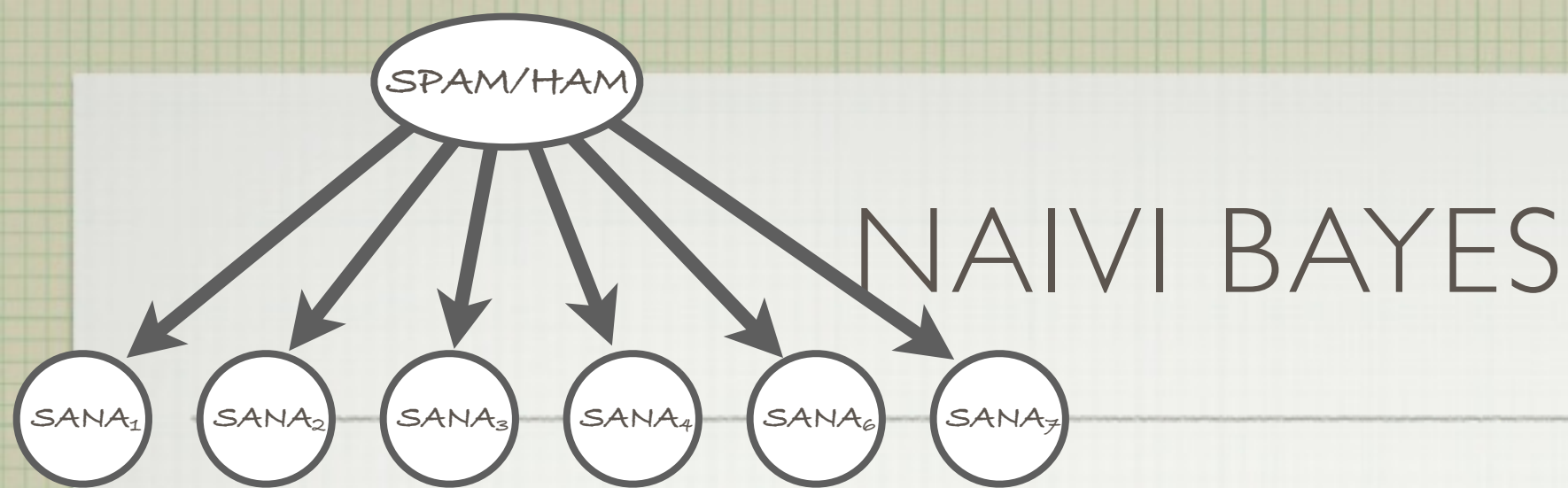
$P(\text{spam} | \text{EVIDENSSI}) / P(\text{ham} | \text{EVIDENSSI})$

$= P(\text{spam}) / P(\text{ham})$

$P(\text{SANA}_1 = \text{viagra} | \text{spam}) / P(\text{SANA}_1 = \text{viagra} | \text{ham})$

$P(\text{SANA}_2 = \text{is} | \text{spam}) / P(\text{SANA}_2 = \text{is} | \text{ham})$

$P(\text{SANA}_3 = \text{algorithm} | \text{spam}) / P(\text{SANA}_3 = \text{algorithm} | \text{ham})$



PSEUDOKOODINA:

SPAMICITY(Viesti, P):

$\text{Odds} = P.\text{Spam} / P.\text{noSpam}$

for each Sana in Viesti

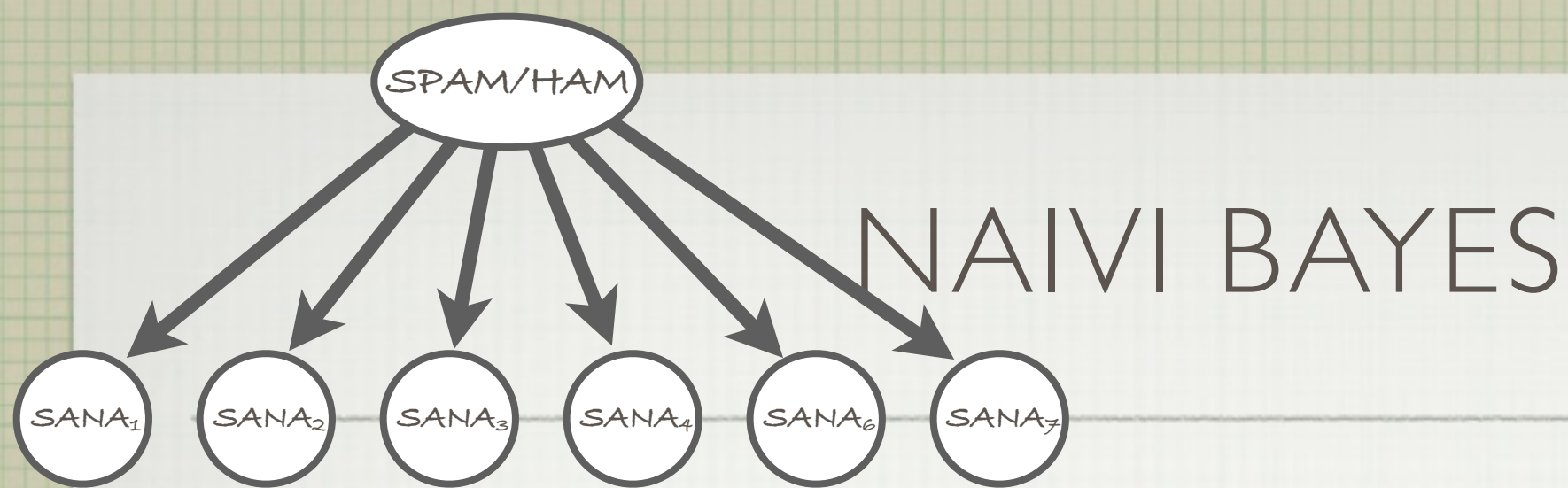
$\text{Odds} = \text{Odds} * P.\text{Sana_Spam}(\text{Sana}) / P.\text{Sana_noSpam}(\text{Sana})$

return(Odds)

JOS SPAMICITY(Viesti, P) >1, LUOKITTELE VIEСТИ SPAMIKSI

JOS SPAMICITY(Viesti, P) <1, LUOKITTELE VIEСТИ HAMIKSI

JOS SPAMICITY(Viesti, P) =1, EN TIEDÄ



PSEUDOKOODINA:

SPAMICITY(Viesti, P):

Odds = $P_{\text{Spam}} / P_{\text{noSpam}}$

for each Sana in Viesti

Odds = Odds * $P_{\text{Sana_Spam}}(\text{Sana}) / P_{\text{Sana_noSpam}}(\text{Sana})$

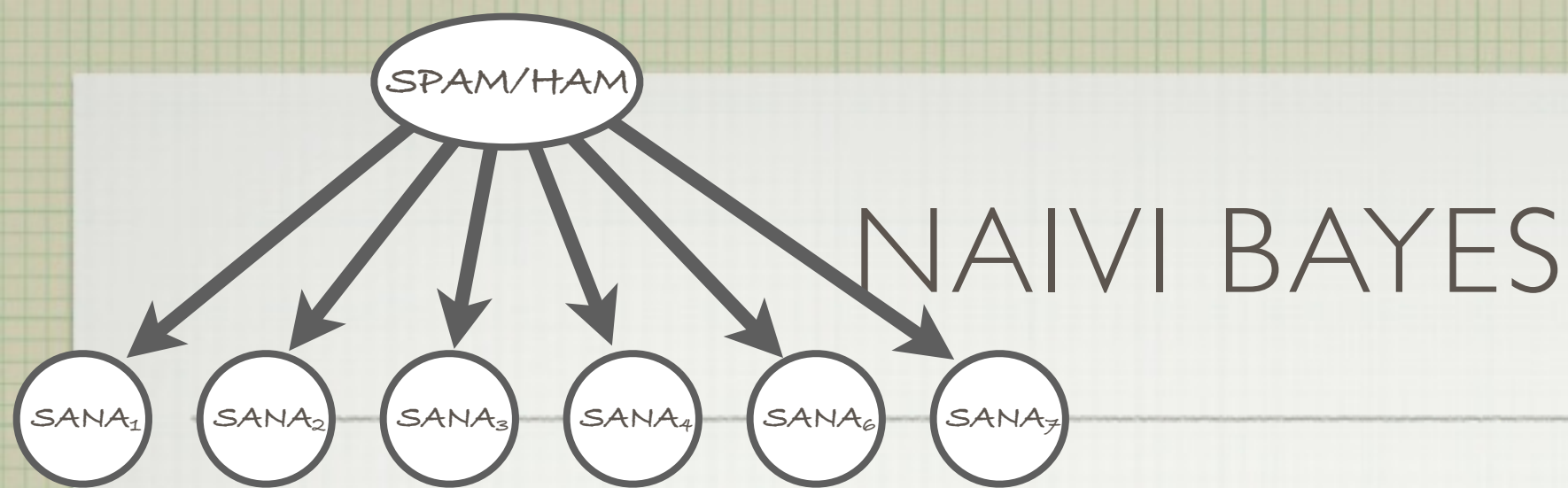
return(Odds)

JOS SPAMICITY(Viesti, P) $> 1 + \alpha$, LUOKITTELE VIEСТИ SPAMIKSI

JOS SPAMICITY(Viesti, P) $< 1 - \beta$, LUOKITTELE VIEСТИ HAMIKSI

MUUTEN, EN TIEDÄ

Epäsymmetrinen kustannusfunktio:
! Asiallisen viestin luokittelu spamiksi
pahempi virhe kuin toisin päin



PSEUDOKOODINA:

$$\text{LOG}(A * B) = \text{LOG}(A) + \text{LOG}(B)$$

SPAMICITY(Viesti, P):

Odds = P.Spam / P.noSpam

for each Sana in Viesti

Odds = Odds * P.Sana_Spam(Sana) / P.Sana_noSpam(Sana)

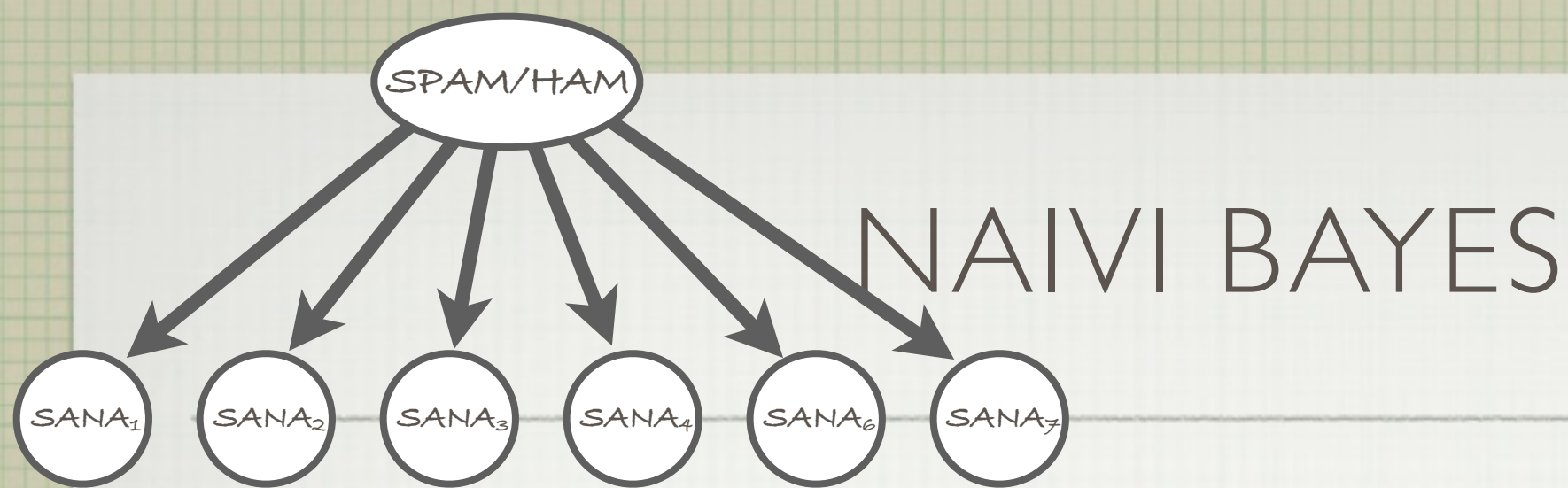
return(Odds)

KÄYTÄNNÖN ONGELMA: **ALI-JA YLIVUODOT**

Odds ARVOSTA TULEE HELPOSTI LIIAN PIENI (LÄHELLÄ NOLLAA)
TAI LIIAN SUURI.

RATKAISU:

KÄYTÄ $\log(\text{Odds})$



PSEUDOKOODINA:

$$\text{LOG}(A * B) = \text{LOG}(A) + \text{LOG}(B)$$

SPAMICITY(Viesti, P):

$\text{logOdds} = \text{log}(\text{P.Spam} / \text{P.noSpam})$

for each Sana in Viesti

$\text{logOdds} = \text{logOdds} + \text{log}(\text{P.Sana_Spam}(\text{Sana}) / \text{P.Sana_noSpam}(\text{Sana}))$

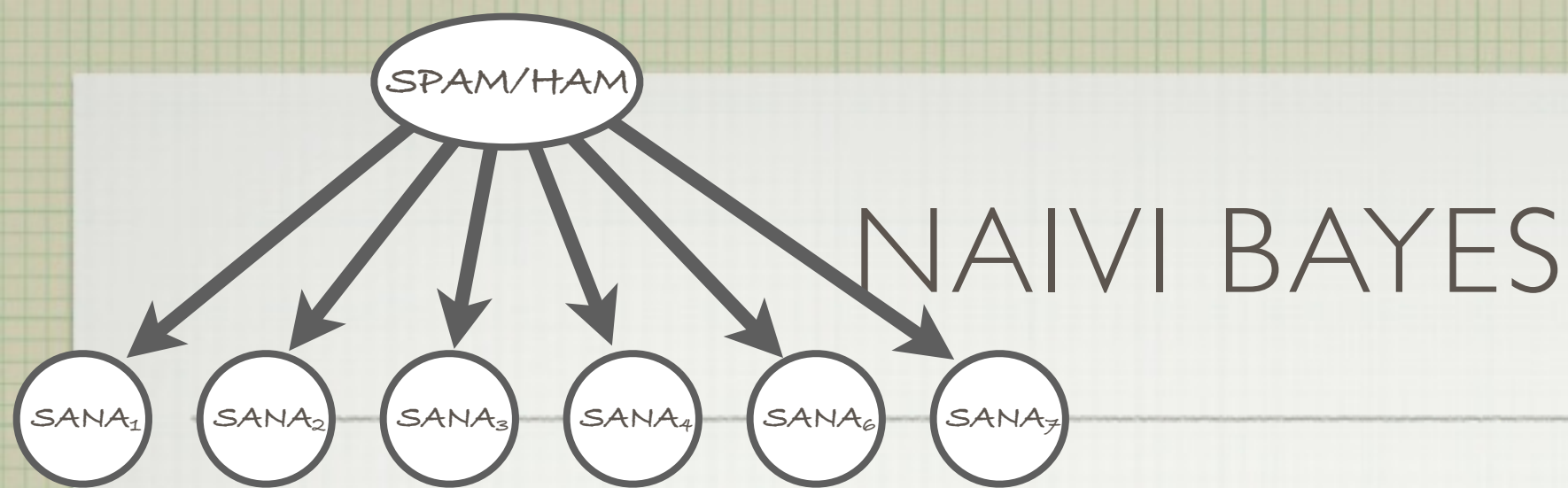
return($\text{exp}(\text{logOdds})$)

KÄYTÄNNÖN ONGELMA: ALI-JA YLIVUODOT

Odds ARVOSTA TULEE HELPOSTI LIIAN PIENI (LÄHELLÄ NOLLAA)
TAI LIIAN SUURI.

RATKAISU:

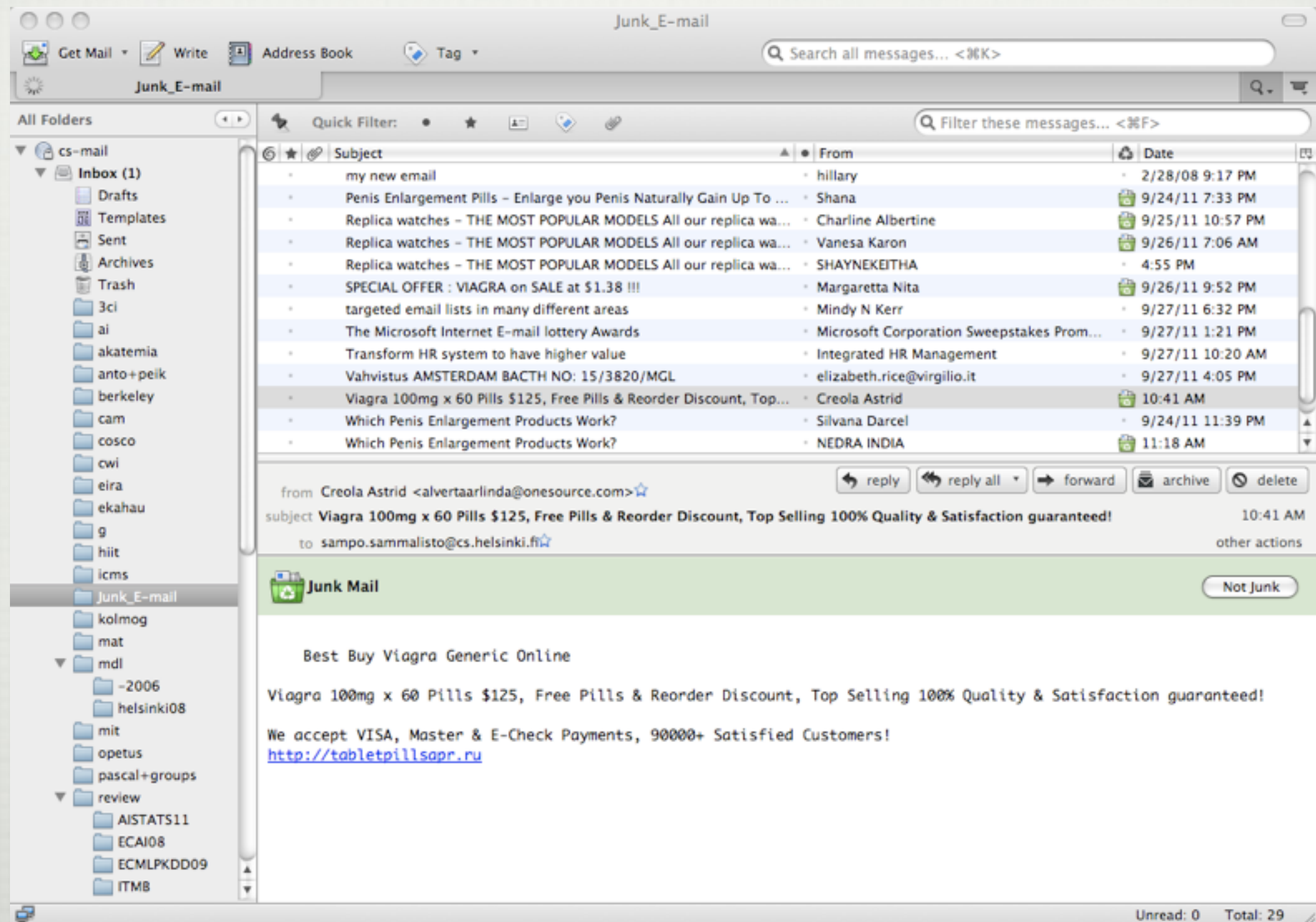
KÄYTÄ $\text{log}(\text{Odds})$



PARAMETRIEN OPPIMISESTA

- * VAIKEA KEKSIÄ PÄÄSTÄ EHDOLLISIA TN:IA.
- * HUONOT ARVOT HUONONTAVAT FILTTERIN TOIMINTAA
- * PAREMPI RATKAISU:
 1. KERÄÄ ISO KASA SPAM-VIESTEJÄ
 2. KERÄÄ ISO KASA HAM-VIESTEJÄ
 3. ARVIOI $P(SANA_i = \underline{\quad}) = 0.\underline{\quad}$ LASKEMALLA DATASTA (VRT. **LASKUHARJ.**)
- * VAROTTAVA NOLLATODENNÄKÖISYYKSIÄ!
(JOS KASA EI TARPEEKSI ISO, JOTKUT SANAT EIVÄT VAIN SATU ESIINTYMÄÄN SIINÄ.)

ESIM.



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ESIM.

SPAM

1	MONEY	0.04	%
...			
5	VIAGRA	0.21	%
...			
10	IS	0.42	%
...			
19	REPLICA	0.80	%
20	EMAIL	0.84	%
20	YOU	0.84	%
21	DATABASE	0.88	%
25	EMAILS	1.05	%
26	OF	1.09	%
31	TO	1.30	%
43	AND	1.80	%
48	THE	2.01	%
TOTAL 2386			

HAM

21	ALGORITHM	0.01	%
...			
62	MONEY	0.02	%
...			
2199	FOR	0.78	%
2492	THAT	0.88	%
2990	YOU	1.05	%
3141	IN	1.11	%
3160	I	1.11	%
3218	AND	1.13	%
3283	IS	1.16	%
3472	OF	1.22	%
3874	A	1.37	%
5442	TO	1.92	%
9196	THE	3.24	%
TOTAL 283736			

ESIM.

SPAM

1	MONEY	0.04	%
...			
5	VIAGRA	0.21	%
...			
10	IS	0.42	%
...			
19	REPLICA	0.80	%
20	EMAIL	0.84	%
20	YOU	0.84	%
21	DATABASE	0.88	%
25	EMAILS	1.05	%
26	OF	1.09	%
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TOTAL 2386			

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3874	A	1.37	%
5442	TO	1.92	%
9196	THE	3.24	%
TOTAL 283736			

ESIM.

SPAM

1	MONEY	0.04	%
...			
5	VIAGRA	0.21	%
...			
10	IS	0.42	%
...			
19	REPLICA	0.80	%
20	EMAIL	0.84	%
20	YOU	0.84	%
21	DATABASE	0.88	%
25	EMAILS	1.05	%
26	OF	1.09	%
31	TO	1.30	%
43	AND	1.80	%
48	THE	2.01	%
TOTAL 2386			

HAM

21	ALGORITHM	0.01	%
...			
62	MONEY	0.02	%
...			
2199	FOR	0.78	%
2492	THAT	0.88	%
2990	YOU	1.05	%
3141	IN	1.11	%
3160	I	1.11	%
3218	AND	1.13	%
3283	IS	1.16	%
3472	OF	1.22	%
3874	A	1.37	%
5442	TO	1.92	%
9196	THE	3.24	%
TOTAL 283736			

ESIM.

SPAM

1 MONEY 0.04 %

...

5 VIAGRA 0.21 %

...

10 I

...

19 R

20 E

20 Y

21 DATABASE 0.88 %

25 EMAILS 1.05 %

26 OF 1.09 %

31 TO 1.30 %

43 AND 1.80 %

48 THE 2.01 %

TOTAL 2386

HAM

21 ALGORITHM 0.01 %

...

62 MONEY 0.02 %

...

$P(SANA_i = \text{MONEY} | \text{SPAM})$

0.0004

----- = ----- = 1.918 > 1

$P(SANA_i = \text{MONEY} | \neg \text{SPAM})$

0.0002



3218 AND 1.13 %

3283 IS 1.16 %

3472 OF 1.22 %

3874 A 1.37 %

5442 TO 1.92 %

9196 THE 3.24 %

TOTAL 283736

ESIM.

SPAM

1 MONEY 0.04 %

...

5 VIAGRA 0.21 %

...

10 I

$P(SANA_i = \text{MONEY} | \text{SPAM})$

0.0004

...

19 R

----- = ----- = 1.918 > 1

20 E

$P(SANA_i = \text{MONEY} | \neg \text{SPAM})$

0.0002

20 Y

21 DATABASE 0.88 %

25 E

$P(SANA_i = \text{IS} | \text{SPAM})$

0.0042

26 O

31 T

----- = ----- = 0.3622 < 1

43 A

$P(SANA_i = \text{IS} | \neg \text{SPAM})$

0.0116

48 T

TOTAL 2386

HAM

21 ALGORITHM 0.01 %

...

62 MONEY 0.02 %

...



3218 AND 1.13 %

TOTAL 283736

ESIM.

SPAM

1	MONEY	0.04	%
...			
5	VIAGRA	0.21	%
...			
10	IS	0.42	%
...			
19	REPLICA	0.80	%
20	EMAIL	0.84	%
20	YOU	0.84	%
21	DATABASE	0.88	%
25	EMAILS	1.05	%
26	OF	1.09	%
31	TO	1.30	%
43	AND	1.80	%
48	THE	2.01	%
TOTAL 2386			

HAM

21	ALGORITHM	0.01	%
...			
62	MONEY	0.02	%
...			
2199	FOR	0.78	%
2492	THAT	0.88	%
2990	YOU	1.05	%
3141	IN	1.11	%
3160	I	1.11	%
3218	AND	1.13	%
3283	IS	1.16	%
3472	OF	1.22	%
3874	A	1.37	%
5442	TO	1.92	%
9196	THE	3.24	%
TOTAL 283736			

YHTEENVETO

YHTEENVETO NAIVI BAYES-SPAMFILTTERISTÄ:

* TARVITAAN:

- "PRIORIJAKAUMA" $P(\text{SPAM}) = 0.$ ___
- "LUOKKAEHDOLLISET" JAKAUMAT

$$P(SANA_i = VIAGRA | SPAM) = 0. \quad P(SANA_i = VIAGRA | \neg SPAM) = 0.$$

$$P(SANA_i = IS | SPAM) = 0. \underline{\hspace{1cm}} \quad P(SANA_i = IS | \neg SPAM) = 0. \underline{\hspace{1cm}}$$

$$P(\text{SANA}_i = \text{ALGORITHM} \mid \text{SPAM}) = 0. \quad P(\text{SANA}_i = \text{ALG.} \mid \neg \text{SPAM}) = 0.$$

- * OLETETAAN ETTÄ $P(\text{SANA}_i | \text{SANA}_j, \text{SPAM}) = P(\text{SANA}_i | \text{SPAM})$
(EHDOLLINEN RIIPPUMATTOMUUS)

YHTEENVETO

(JATKOA...):

- * KUSTANNUSFUNKTIO **EPÄSYMMETRINEN**: PAREMPI SÄÄSTÄÄ MUUTAMA SPAM KUIN HUKATA OIKEA Viesti (HAM)
- * JAKAUMAT PARAS **ESTIMOIDA** DATASTA